

SCUTOIDS

between two parallel planes

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Abstract

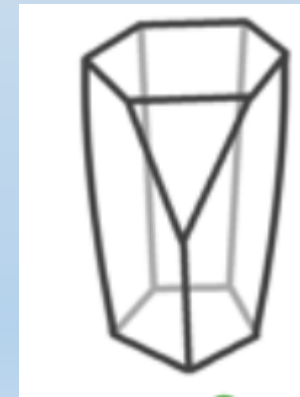
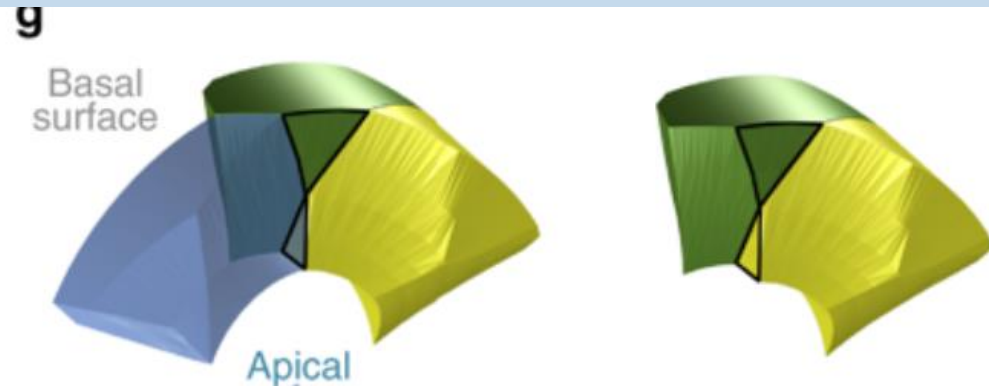
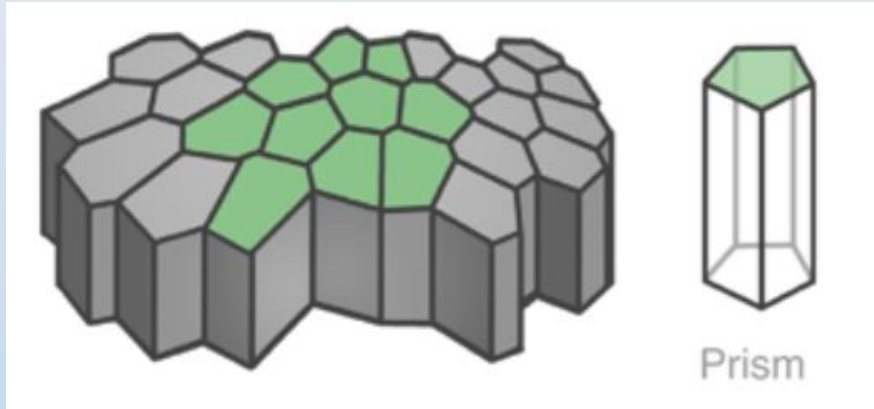
At the end of July 2018, researchers from the University of Seville and Lehigh University (USA) published an article titled »Scutoids are a geometric solution to three-dimensional packing of epithelia«. The solution to the problem of packing epithelia cells in curved shapes was the discovery of a new geometric shape – the scutoid. The scutoid is a solid similar to the prism, whose base surfaces are two different parallel n -gons, the side edges are line segments or some other suitable curves of which at least two form the connection in the shape of the letter Y. That is necessary because of the different number of vertices in the base surfaces. The surfaces of the scutoid can be curved, which enables them to be used for the modelling of the way cells in the epithelial tissues connect.

In our research project, scutoids were observed in spaces between two parallel planes and were analysed from mathematical aspects. The definition was limited to those scutoids whose basic surfaces are the correct polygons with n and $n + 1$ vertices, which are connected with line segments. We searched for those scutoids which supplement each other in pairs alongside the surfaces surrounding Y. In the sources available, the only scutoid described is the one with a pentagon and a hexagon as its base surfaces, which we named the complementary 5-6 scutoid. The aim of the research project was to design and analyse the complementary 5-6 scutoid. In addition, two other scutoids were constructed, the complementary scutoid 4-5 and the complementary scutoid 3-4. Their properties were described and the way in which they connect in pairs was researched.

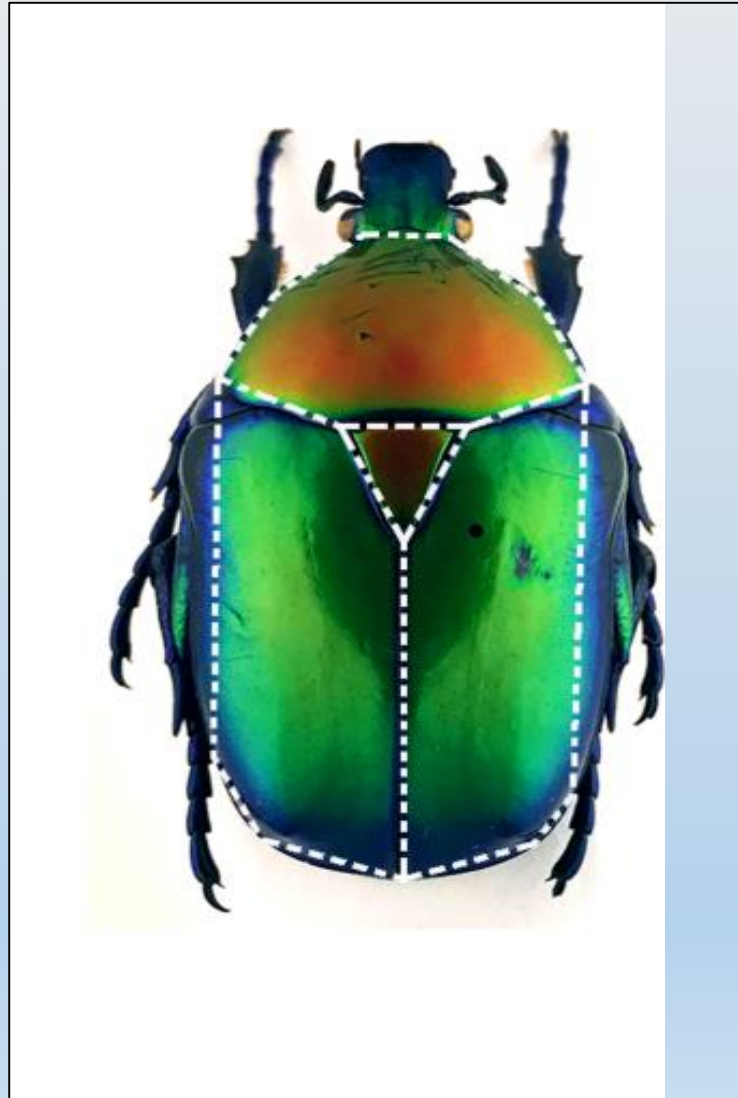
Key words: scutoid, scutellum, space packing, epithelia.

INTRODUCTION= CHALLENGE

»Scutoids are a geometric solution to three-dimensional packing of epithelia«
(<https://www.nature.com/articles/s41467-018-05376-1>). Journal: Nature, July 27th 2018



Name?



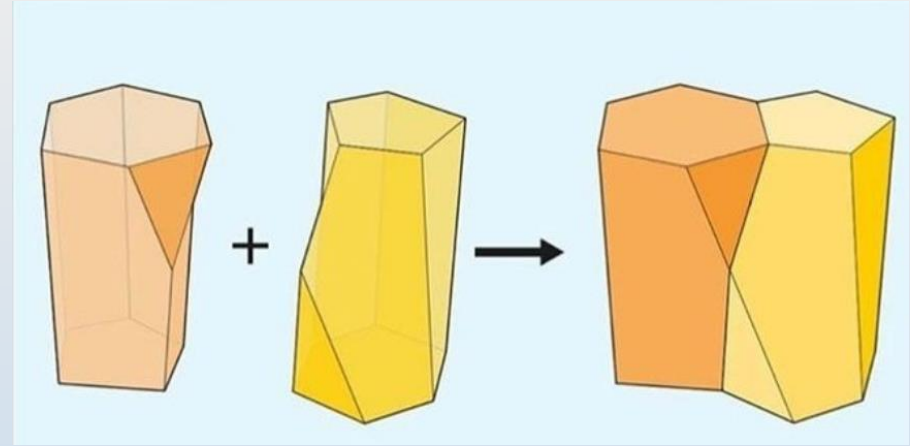
Bugs *Cetoniidae* have
scutellum

The definition of scutoid (with parallel bases)

The scutoid is a solid similar to the prism, whose base surfaces are two different parallel n -gons, the side edges are line segments or some other suitable curves of which at least two form the connection in the shape of the letter Y. The surfaces of the scutoid, except the base and *scutellum*, can be curved.



Aims:



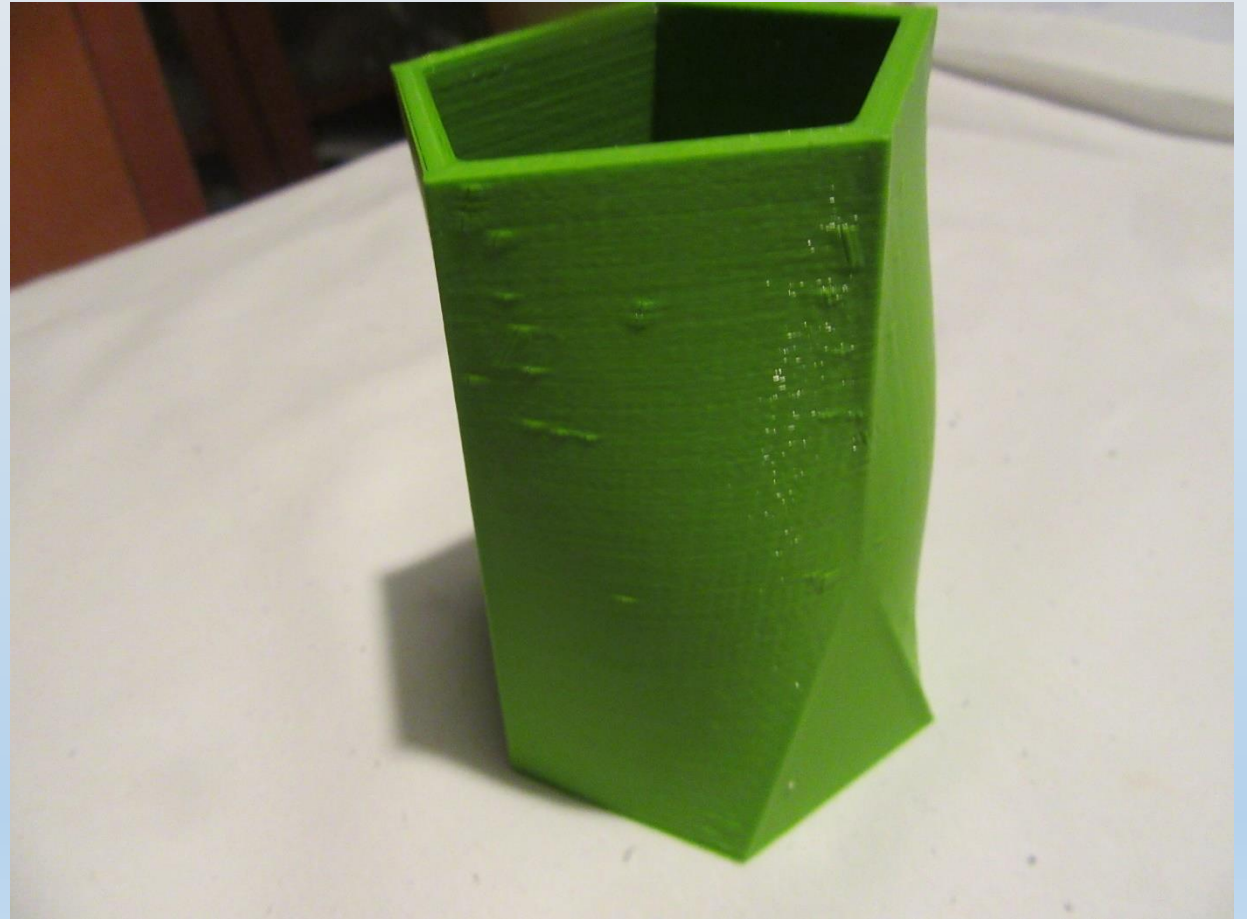
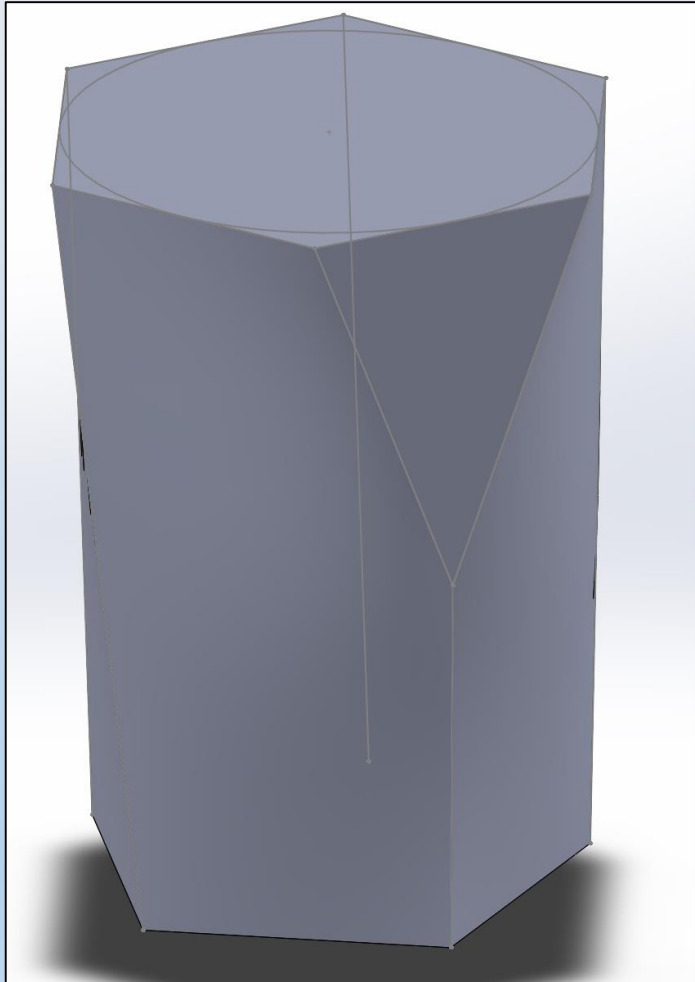
- to design the complementary 5-6 scutoid
- to describe the properties of the scutoid
- to design the complementary scutoid 4-5 and 3-4
- to put forward as many problems as possible and find solutions for them

Methods:

- work according to the original article and internet sources
- forming hypotheses using 3D models, constructed in programme *SOLIDWORKS* and printed by 3D printer
- mathematical calculations and proving the hypotheses

RESULTS:

ORDINARY SCUTOID 5-6



Ordinary scutoid 5-6 – properties:

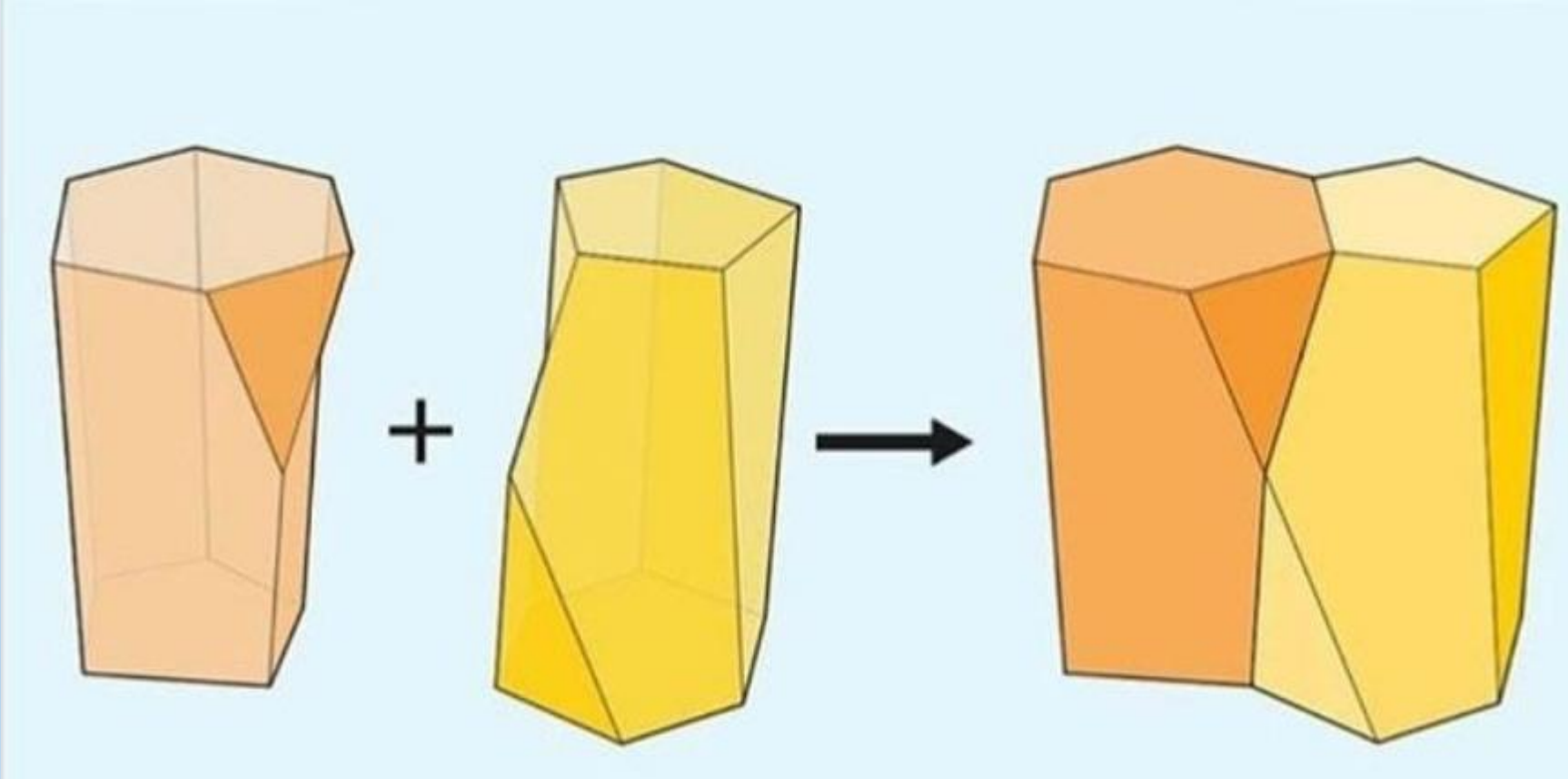
- legs Y lie in the same plane,
- it does not supplement each other in pairs alongside the surfaces surrounding Y .

Ordinary scutoid 5-6 – connecting lx

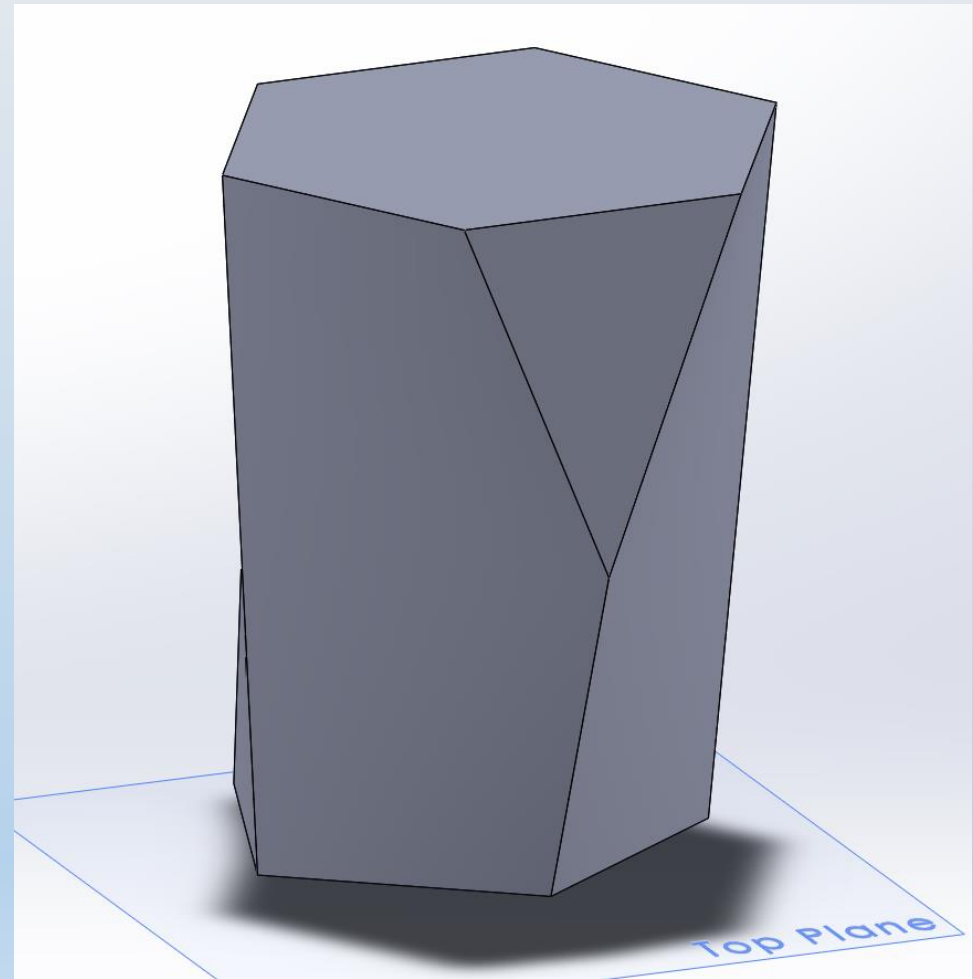
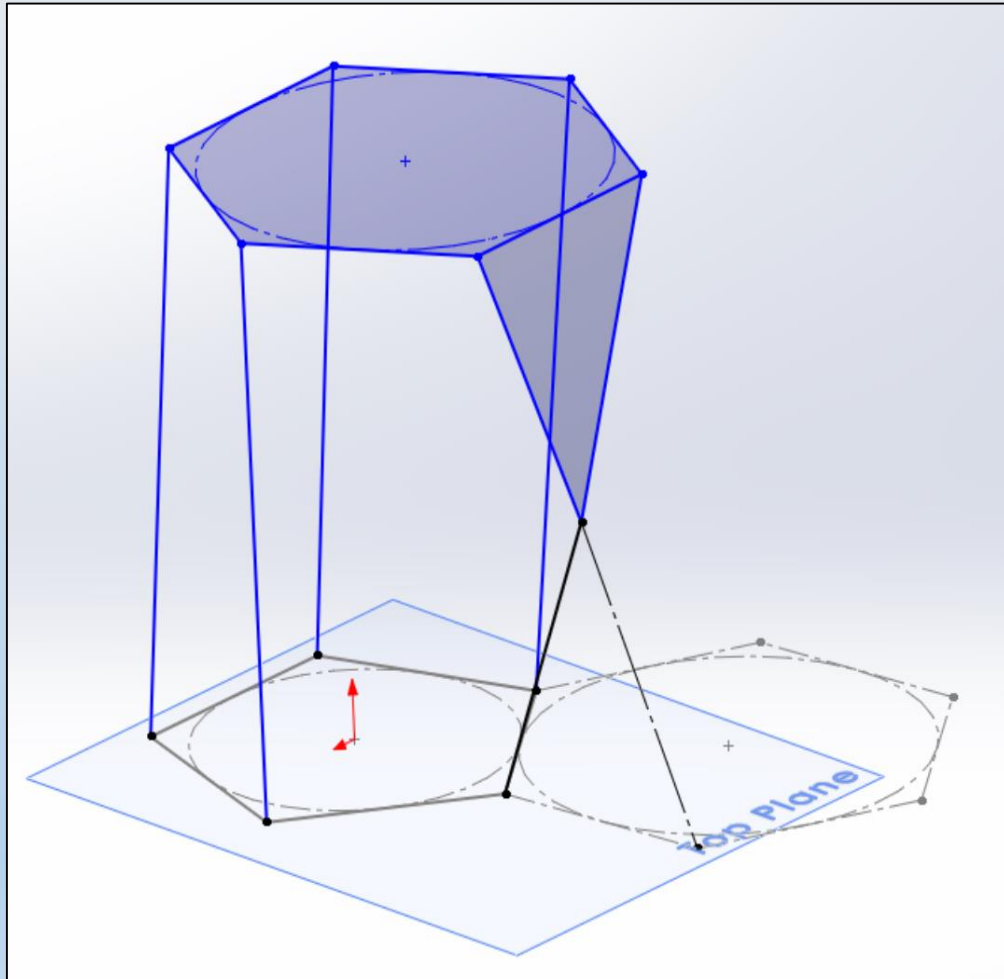


Complementary scutoid 5-6

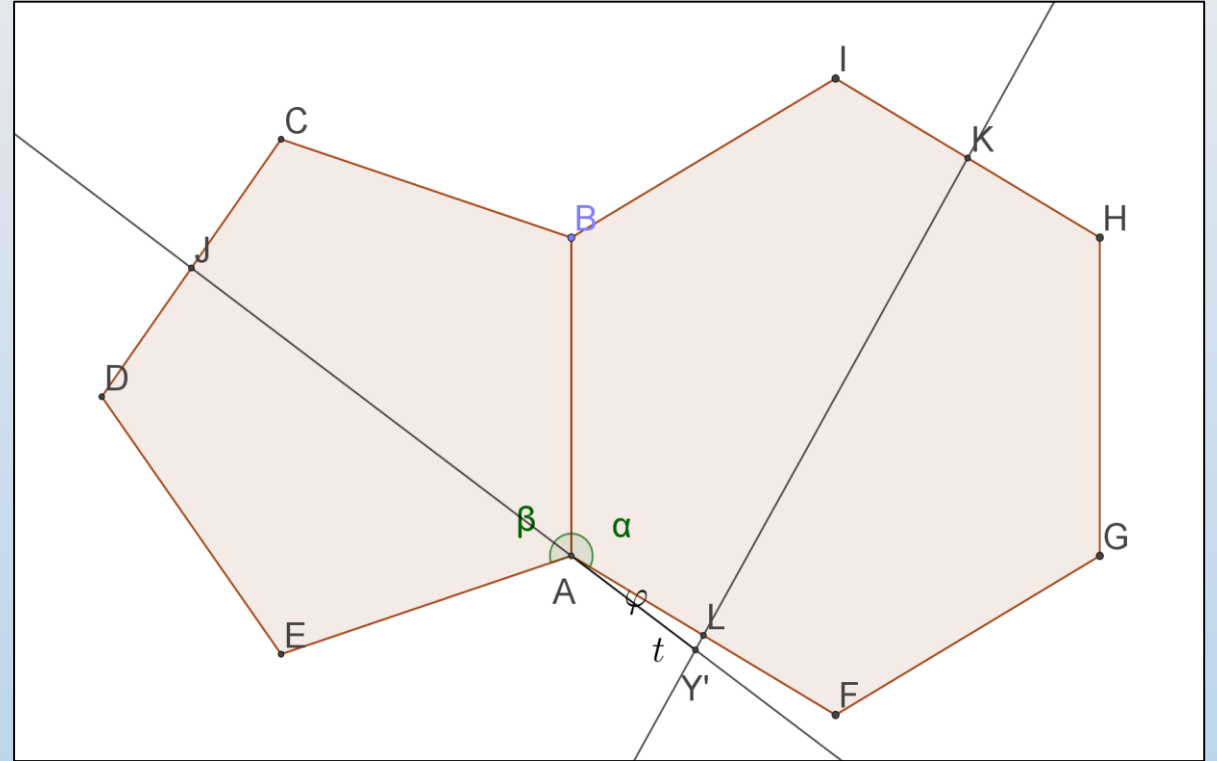
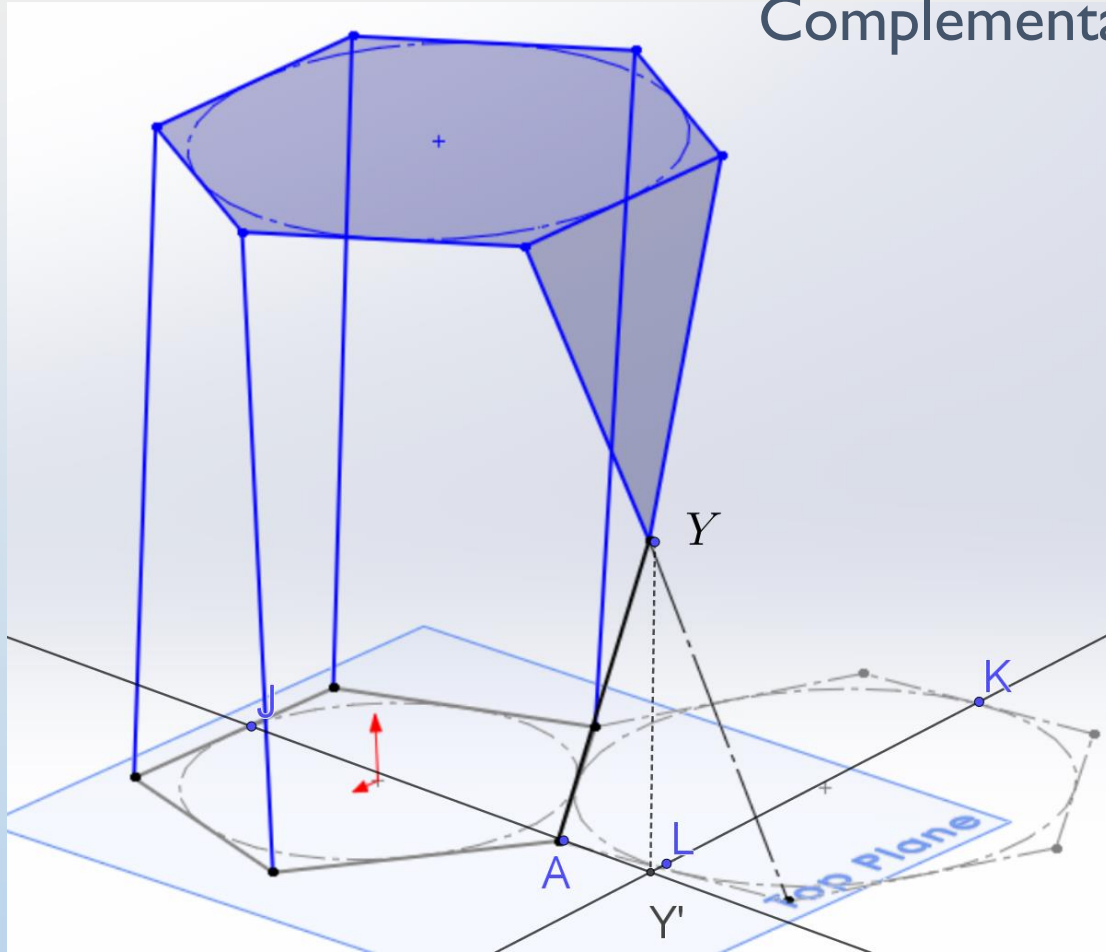
(Pair of Packable scutoids)



Complementary scutoid 5-6: construction

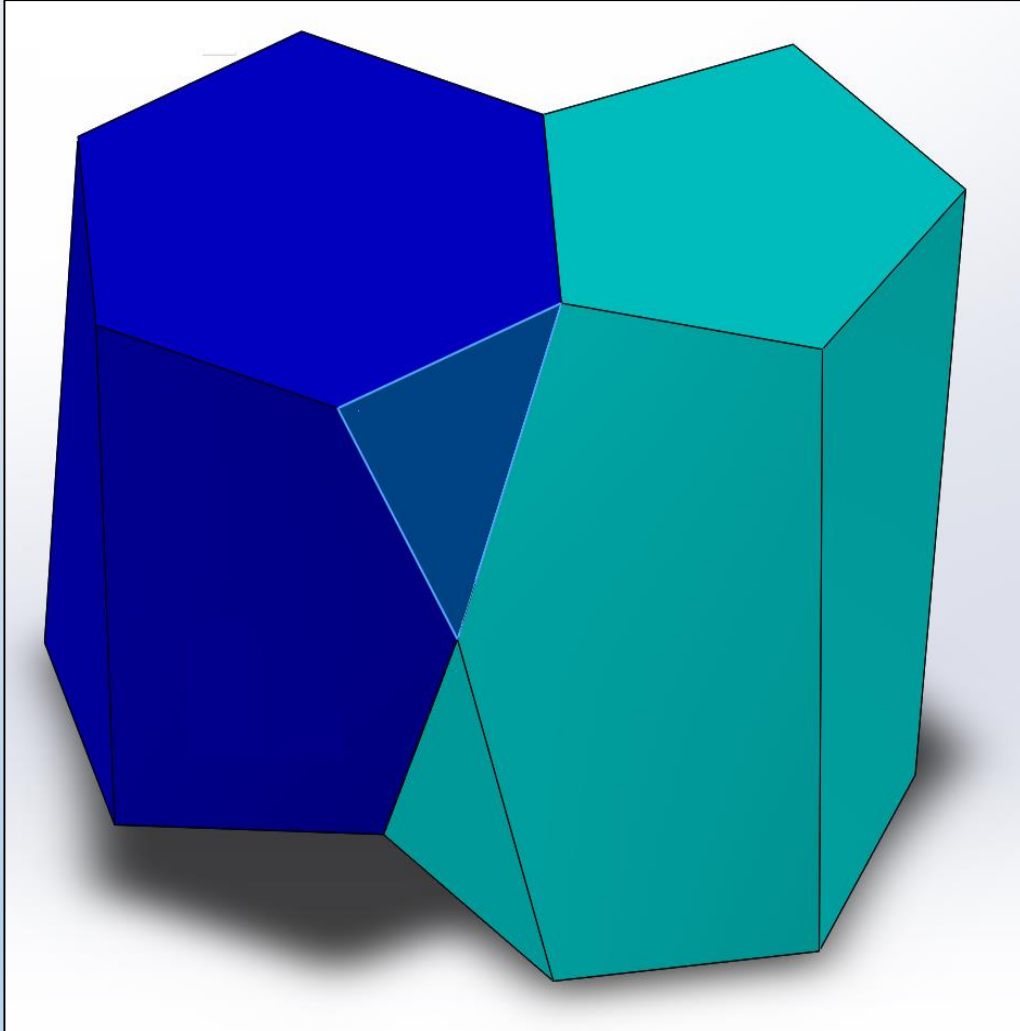


Complementary scutoid 5-6: the length of the leg of scutellum



We calculate the angle $\varphi = 180^\circ - \alpha - \frac{\beta}{2} = 6^\circ$, length of $t = \frac{a}{2 \cdot \cos \varphi}$
 (AY'L), and in the triangle AY'Y use the Pythagorean theorem: $AY = x, AY' = t, Y'Y = \frac{h}{2}$

Complementary scutoid 5-6: the length of the leg of scutellum



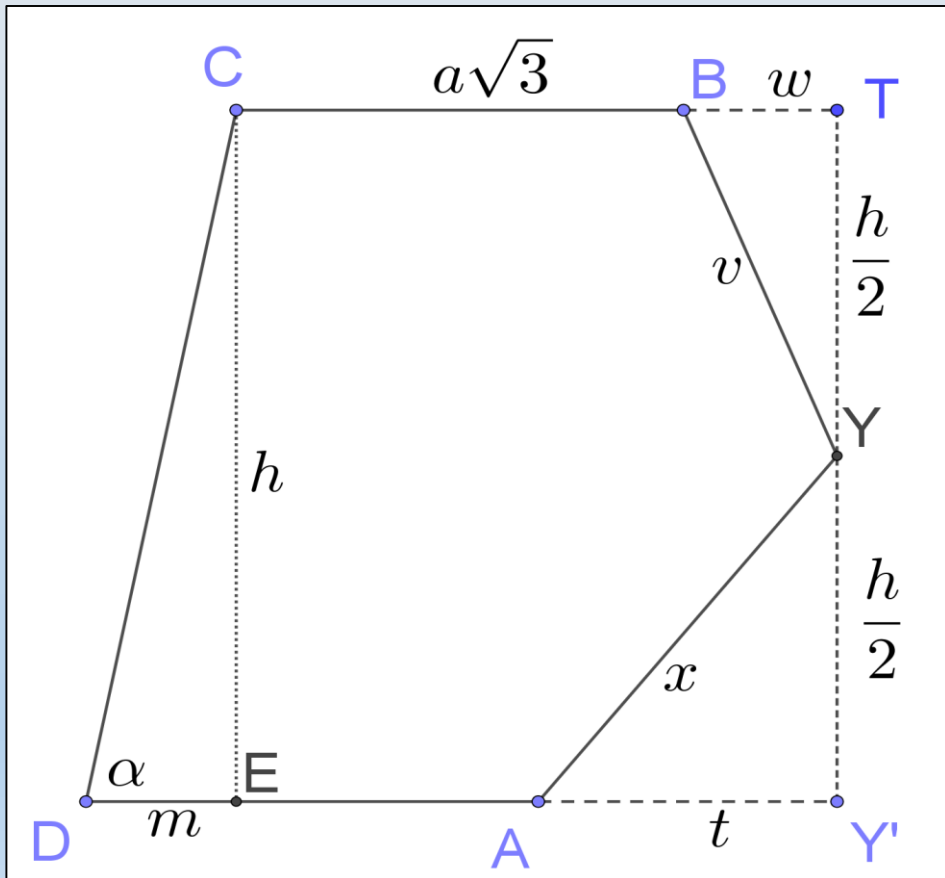
$$x^2 = \left(\frac{h}{2}\right)^2 + t^2$$

the leg of scutellum = x =

$$= \frac{\sqrt{2} \cdot \sqrt{h^2 \cdot (1 + \cos 12^\circ) + 2a^2}}{4 \cdot \cos 6^\circ}$$

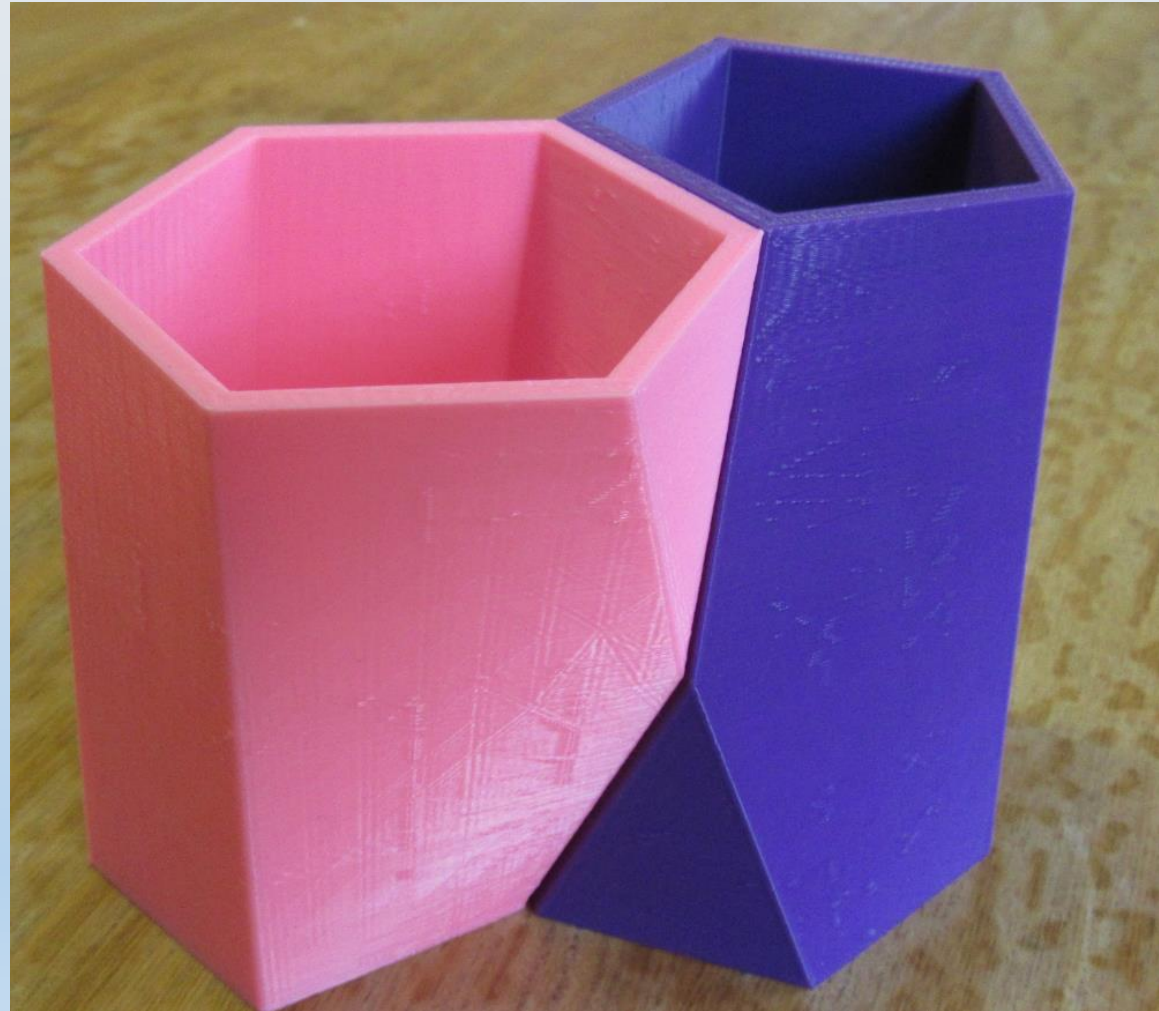
Complementary scutoid 5-6: the angle of inclination of the back surface

$$\tan \alpha = \frac{2h(\sin 6^\circ + 1)}{a(\cos 6^\circ + (\sqrt{2\sqrt{5} + 5} - 2\sqrt{3})(\sin 6^\circ + 1))}$$



The model of complementary scutoid 5-6 the length of side $a = 3.5 \text{ cm}$ and height $h = 8.8 \text{ cm}$. In this case the angle of inclination equals $\alpha = 84.16388769^\circ$.

Complementary scutoid 5-6- connecting 3x



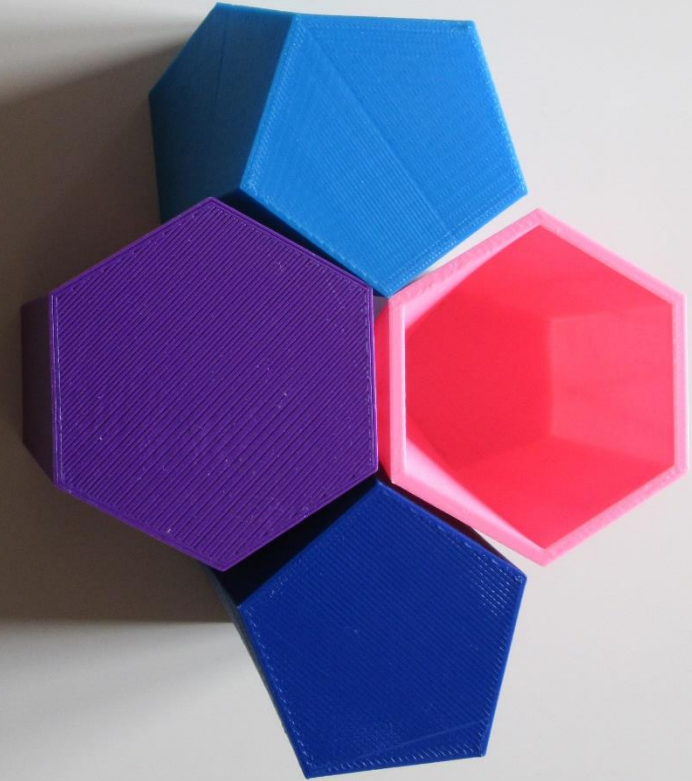
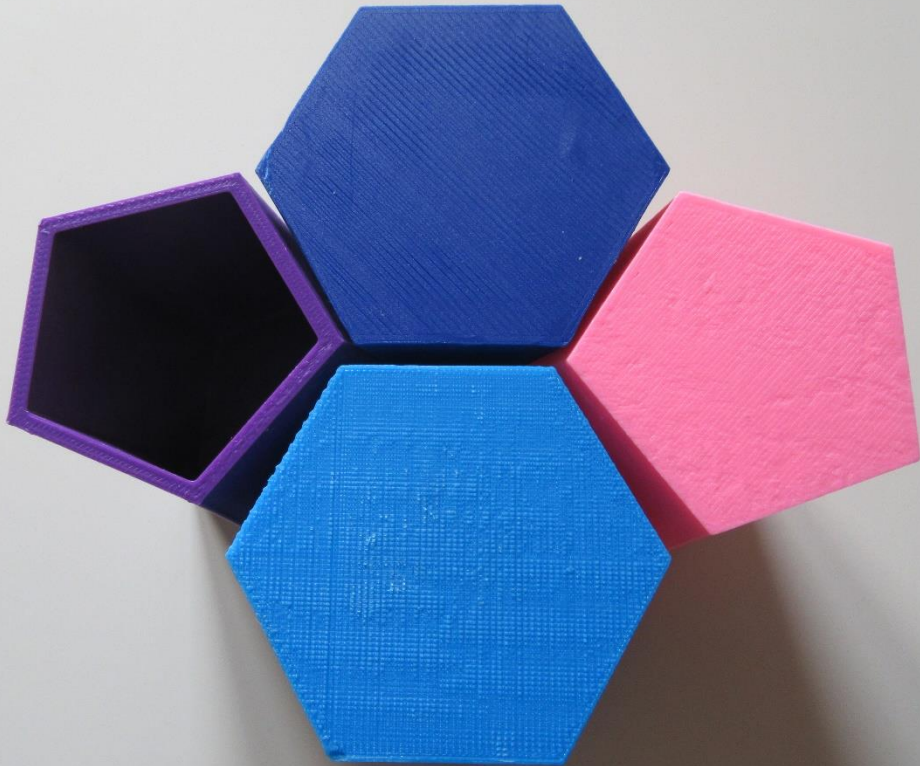


We can't construct the circle, because the equation $k \cdot 48 = 360$ is not solvable in N .

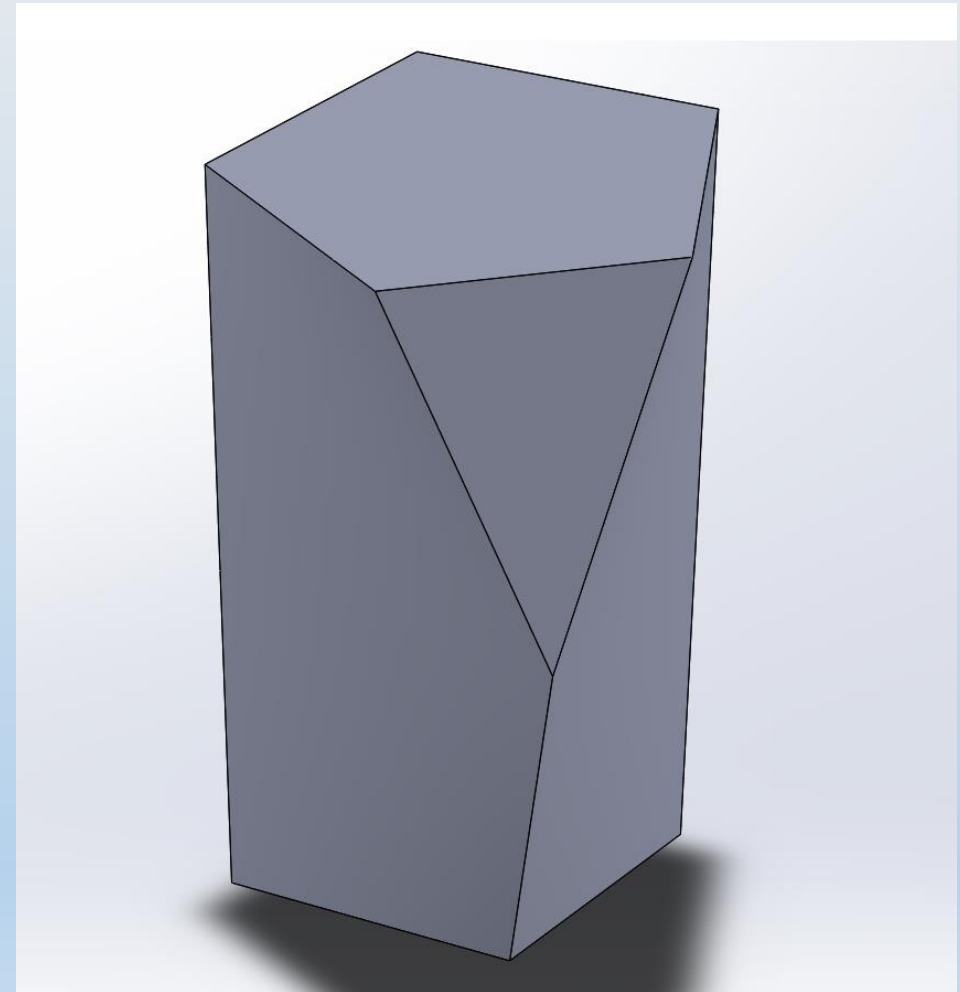
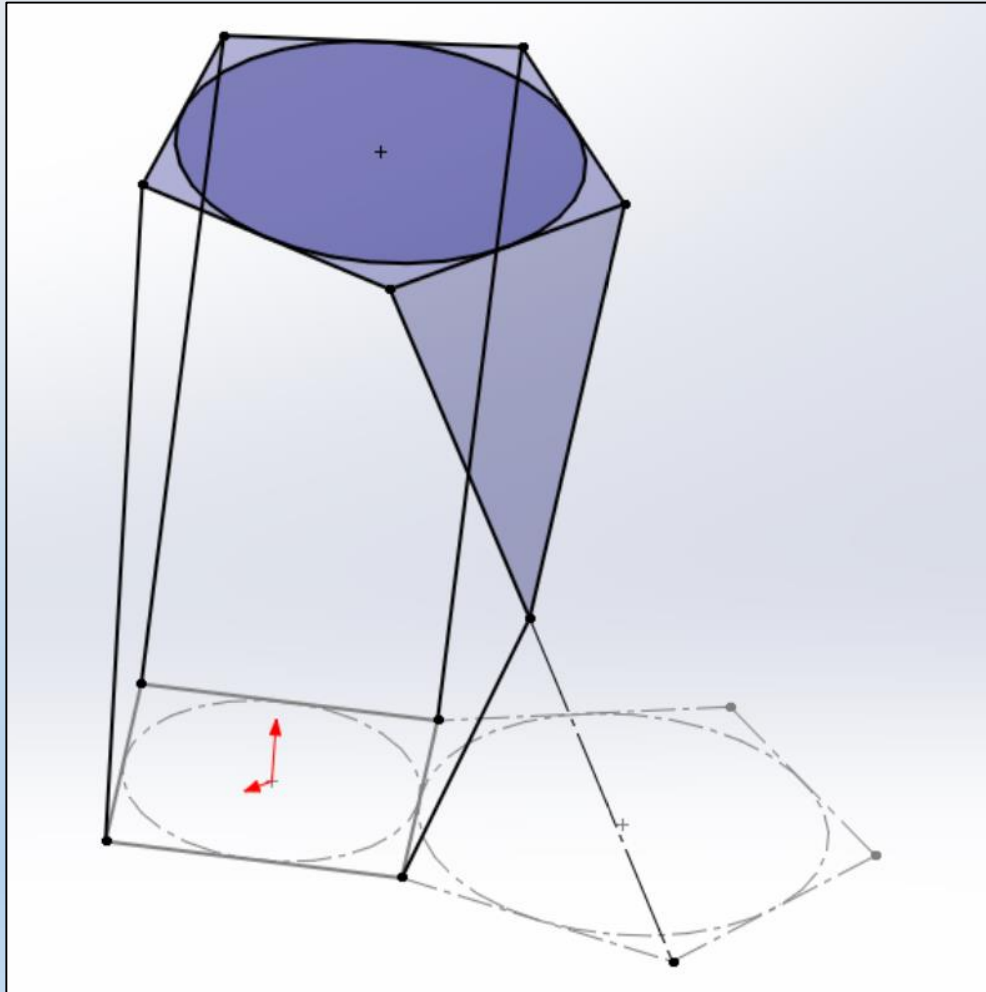




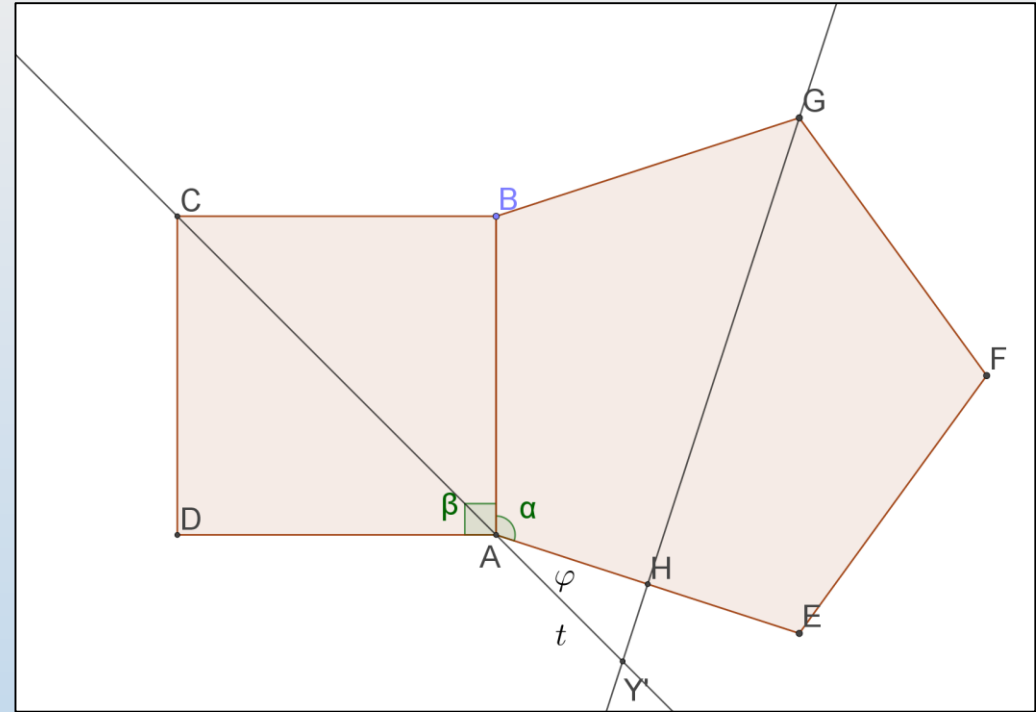
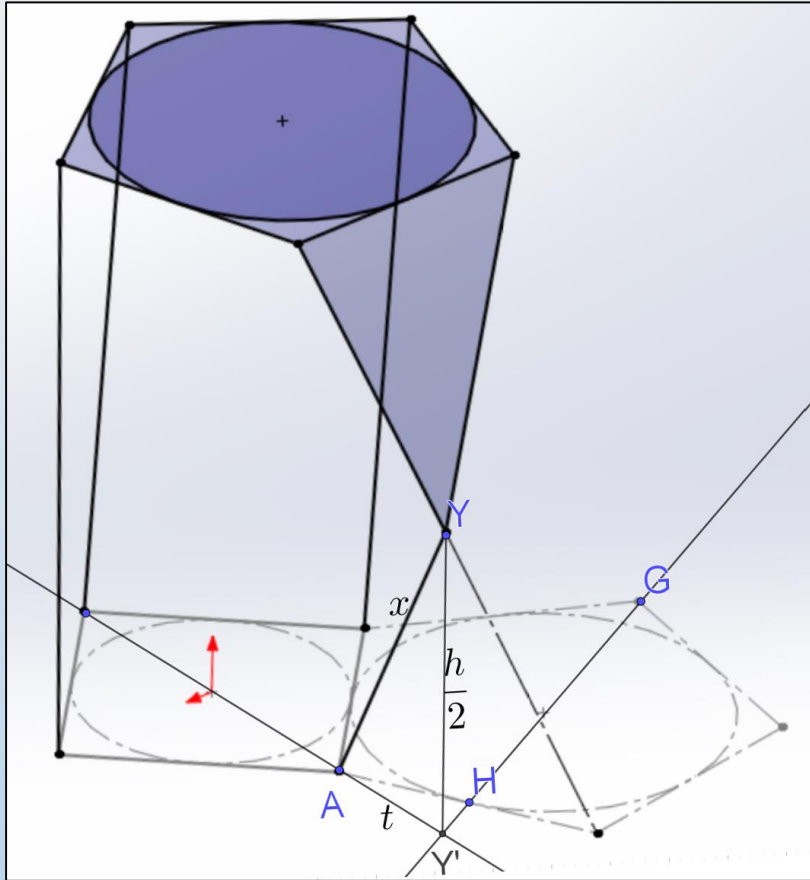
The view from above (left) and from below (right)
→ Different neighbours



Complementary scutoid 4-5 – construction SOLIDWORKS



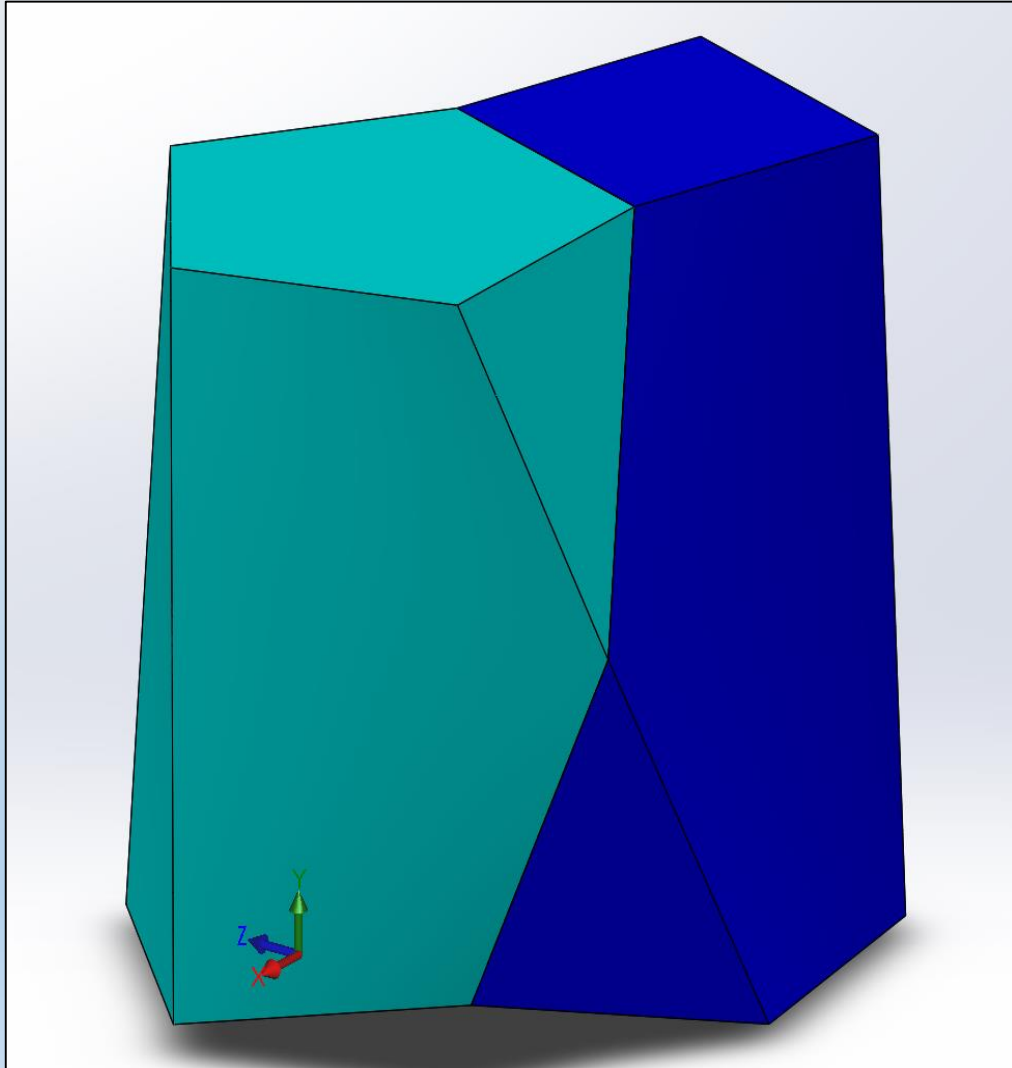
Complementary scutoid 4-5: the lenght of the leg of scutellum



$$\varphi = 180^0 - \alpha - \frac{\beta}{2} = 27^0$$

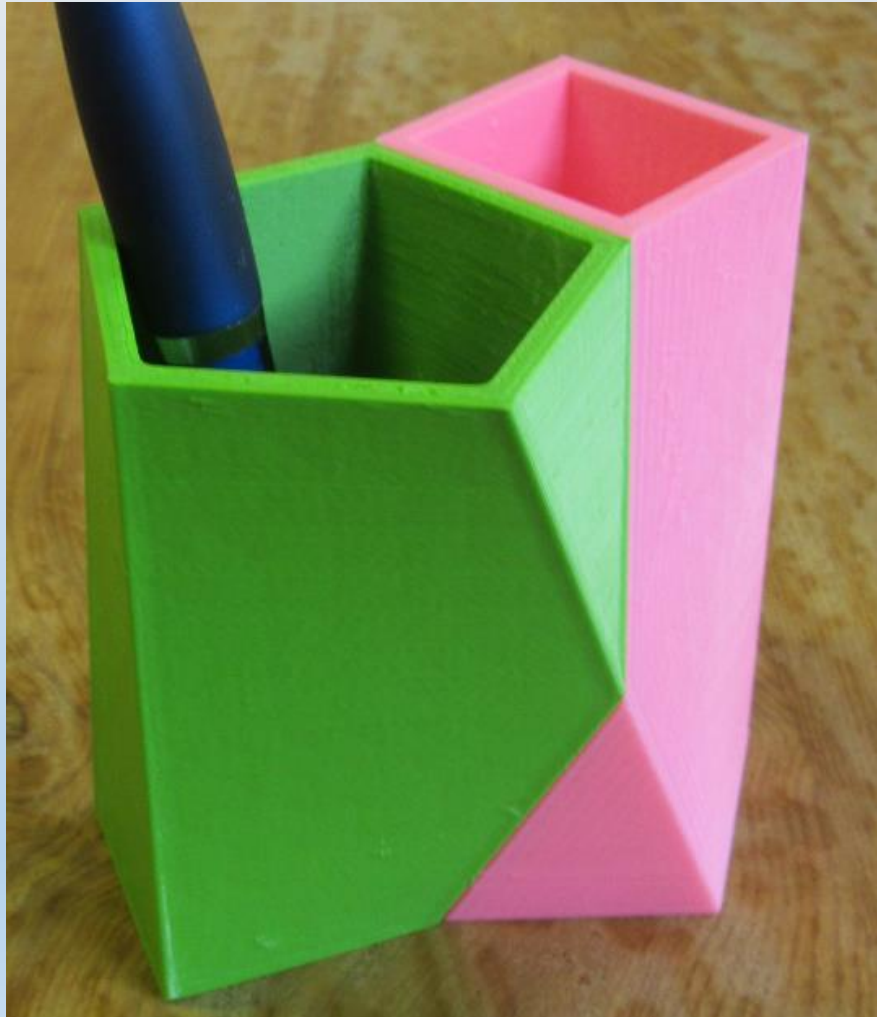
$$\text{the leg of scutellum} = x = \frac{\sqrt{h^2 - 2a^2 \left(\sqrt{(50 - 22\sqrt{5})} + 2\sqrt{5} - 6 \right)}}{2}$$

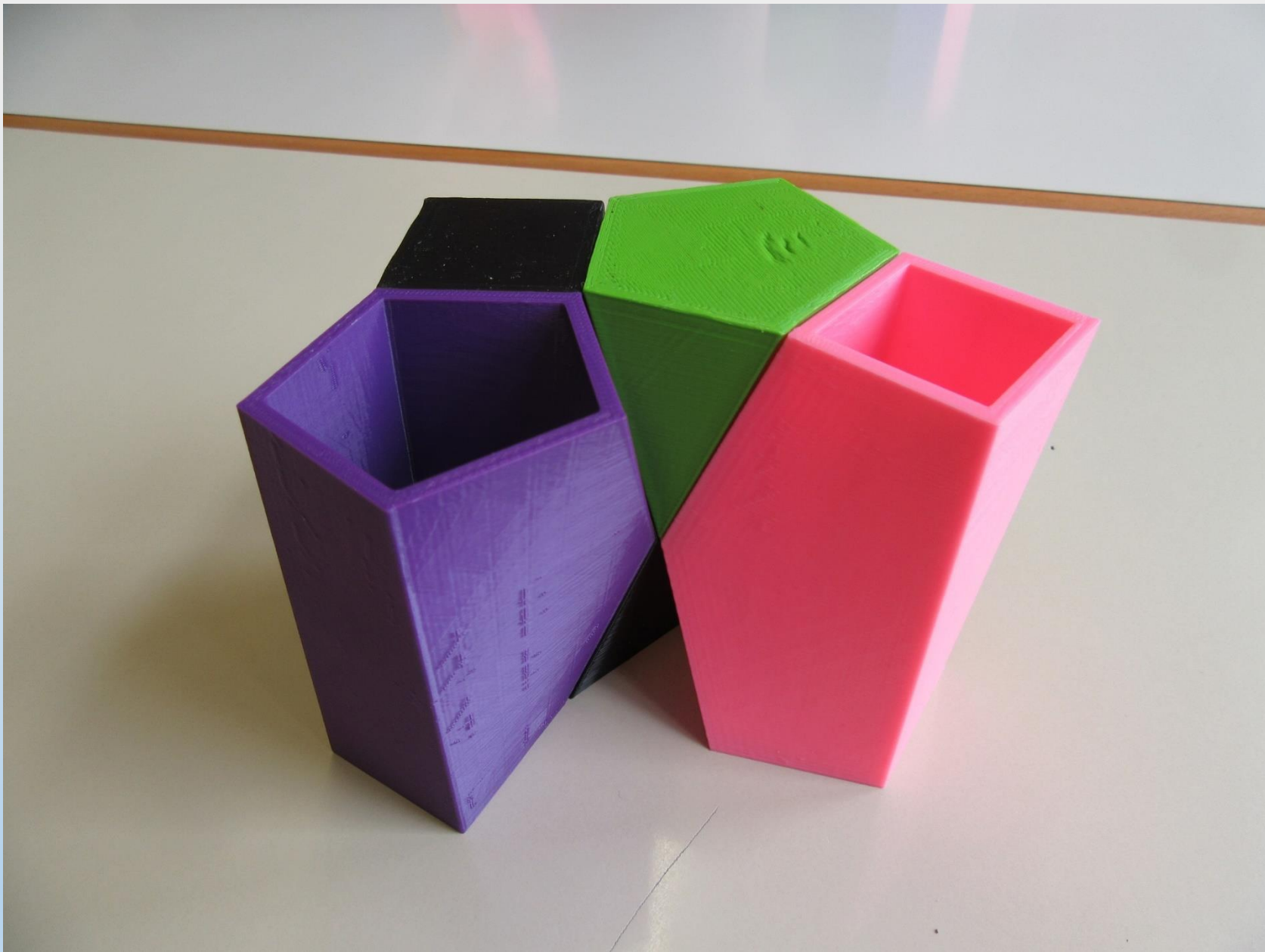
Complementary scutoid 4-5: doesn't have the back surface



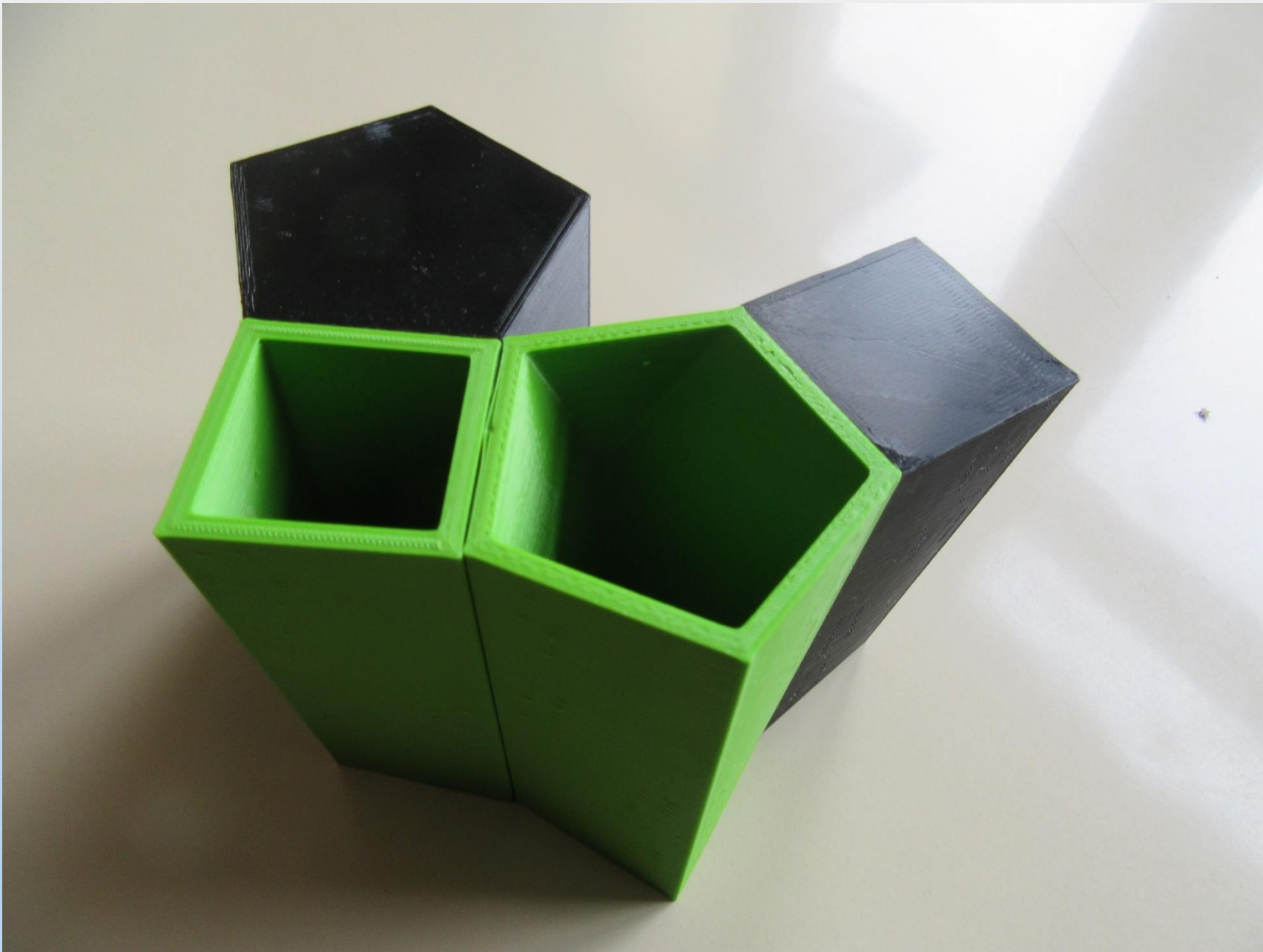
Doesn't have the back surface, opposite Y is the side edge

Complementary scutoid 4-5: connecting 2x



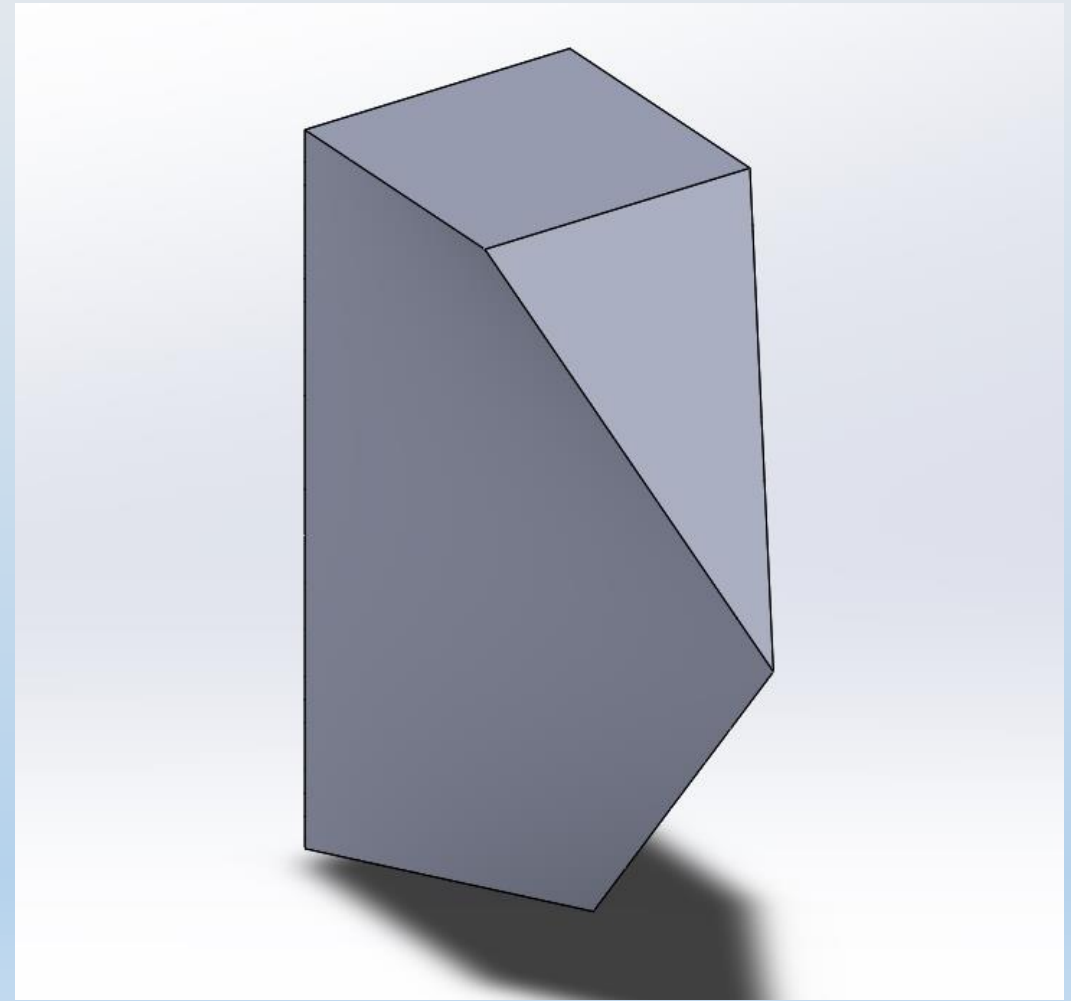
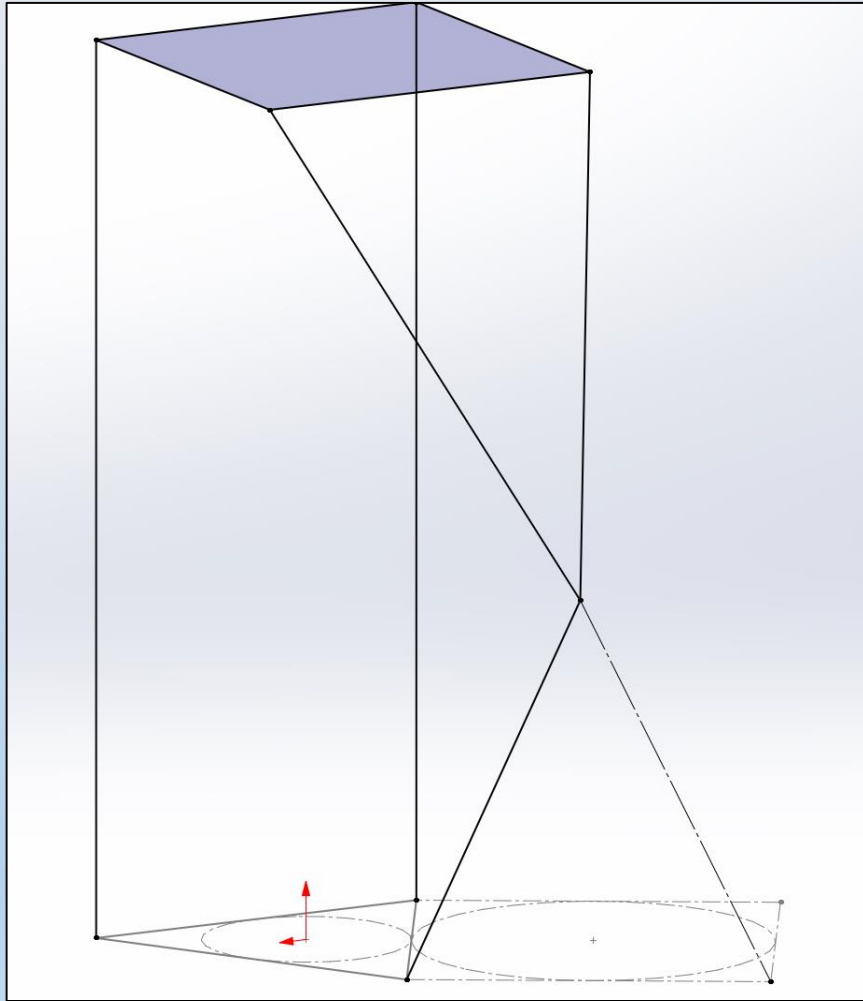


One after another we can compose only four complementary scutoids 4-5

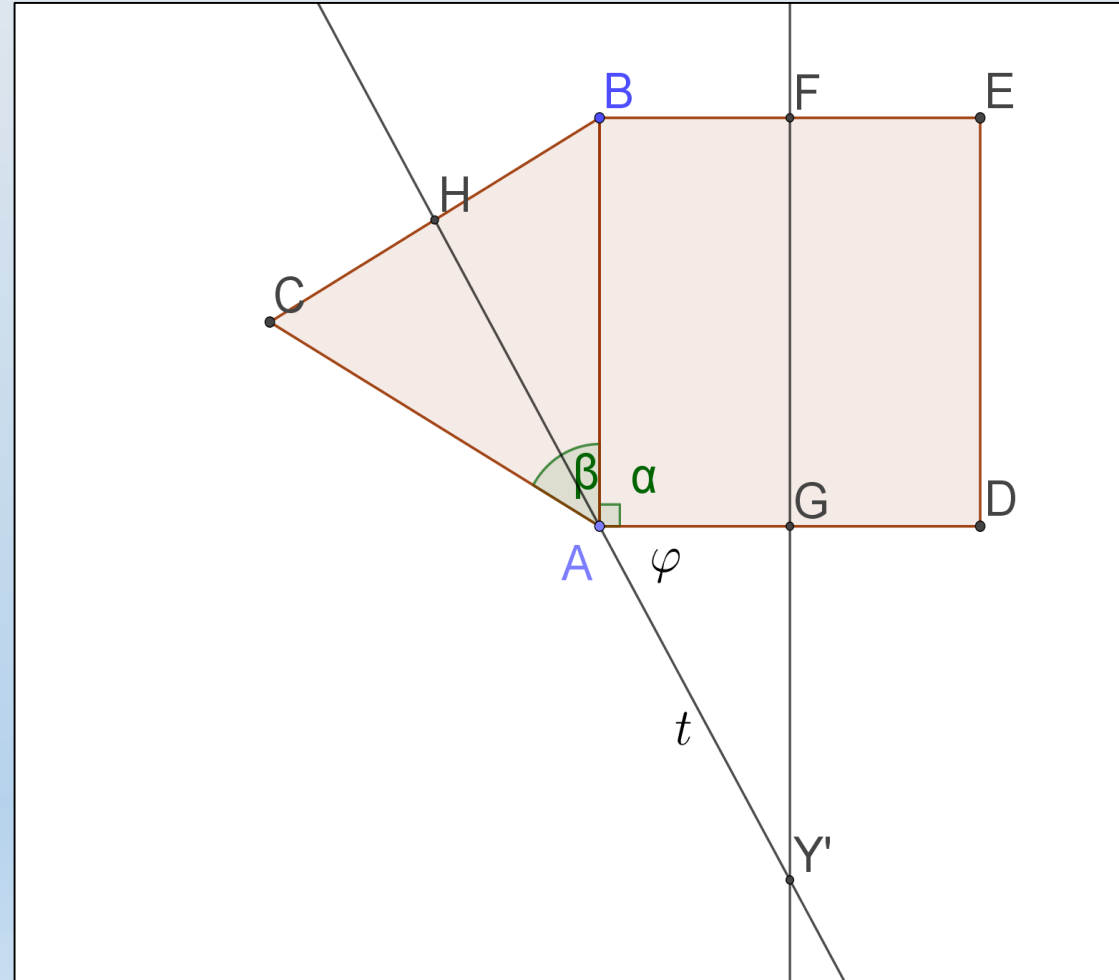
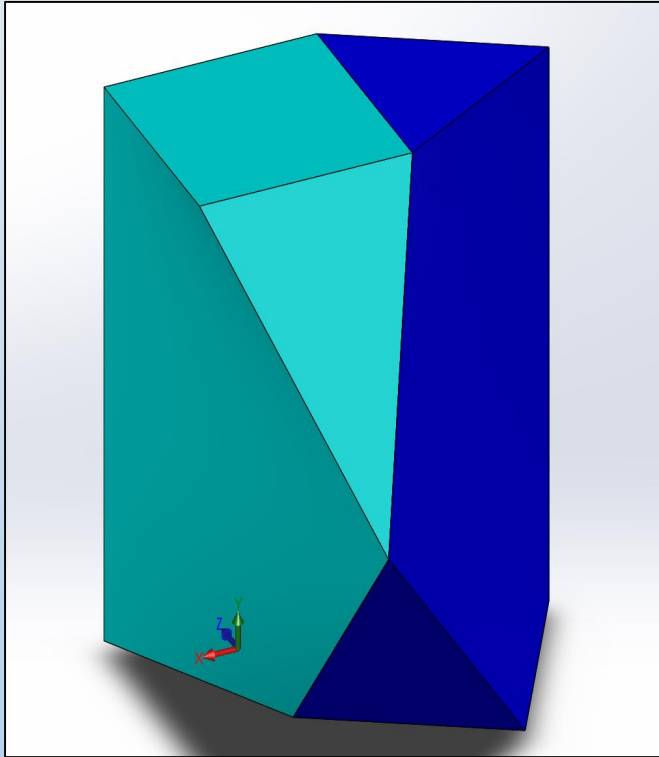


We can not form the circle. The equation $k \cdot 18 = 360$ is in \mathbb{N} solvable, but we can not compose more than four scutoids.

Complementary scutoid 3-4



Complementary scutoid 3-4: the length of the leg of the scutellum

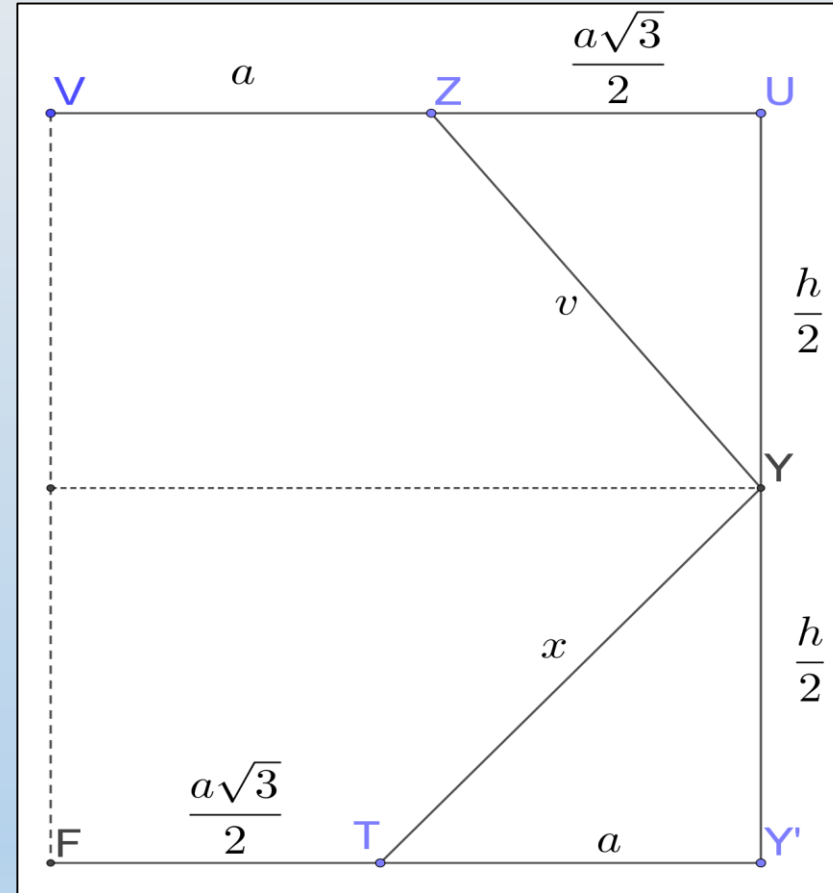
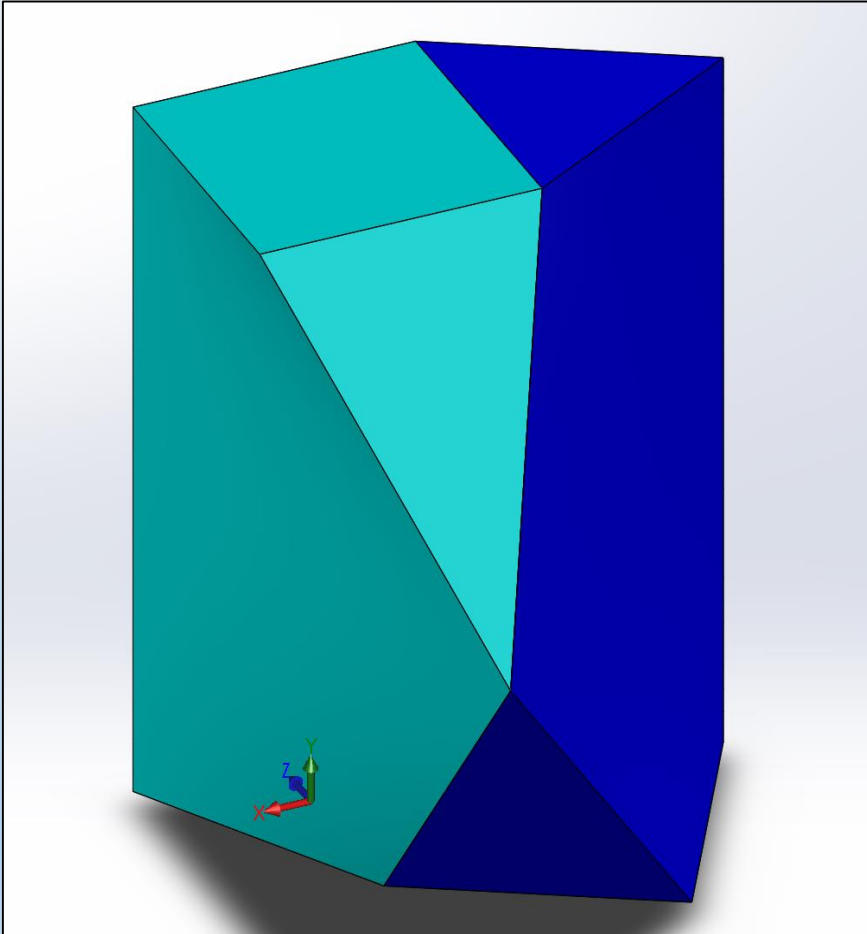


$$\varphi = 180^\circ - \alpha - \frac{\beta}{2} = 60^\circ$$

The leg of scutellum =

$$= \frac{1}{2} \sqrt{(4a^2 + h^2)}$$

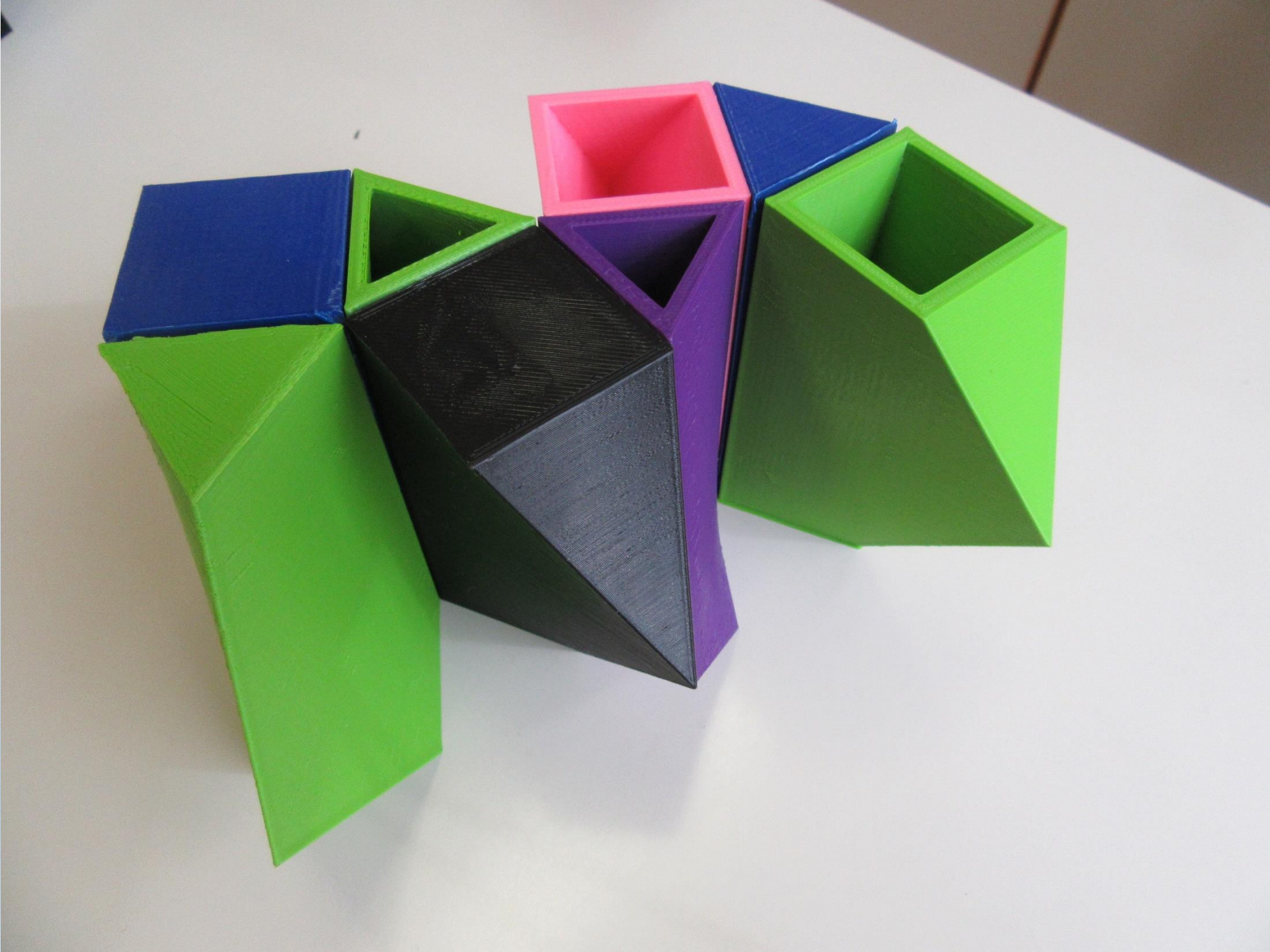
Complementary scutoid 3-4: the angle of inclination of the back surface

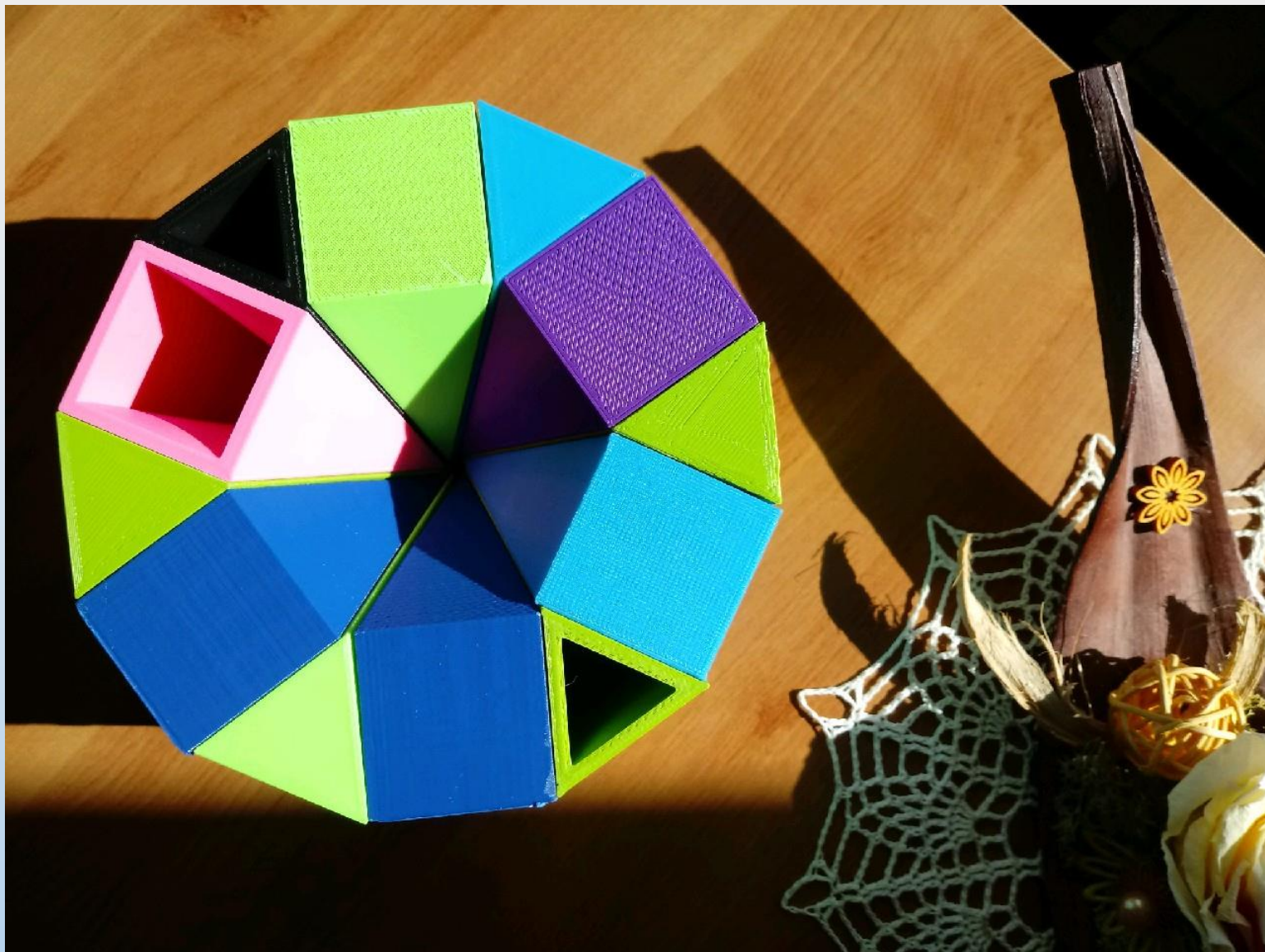


The back surface is perpendicular to the plane of the back surface

Complementary scutoid 3-4: connecting 4x

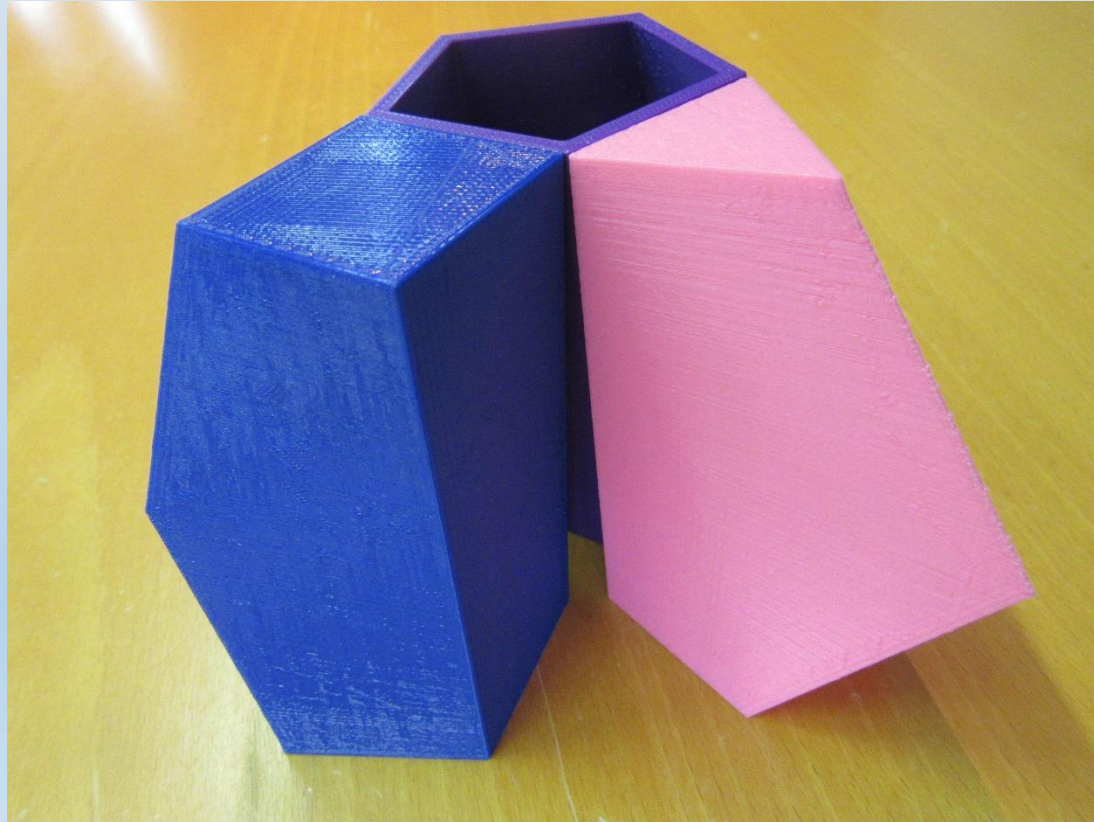




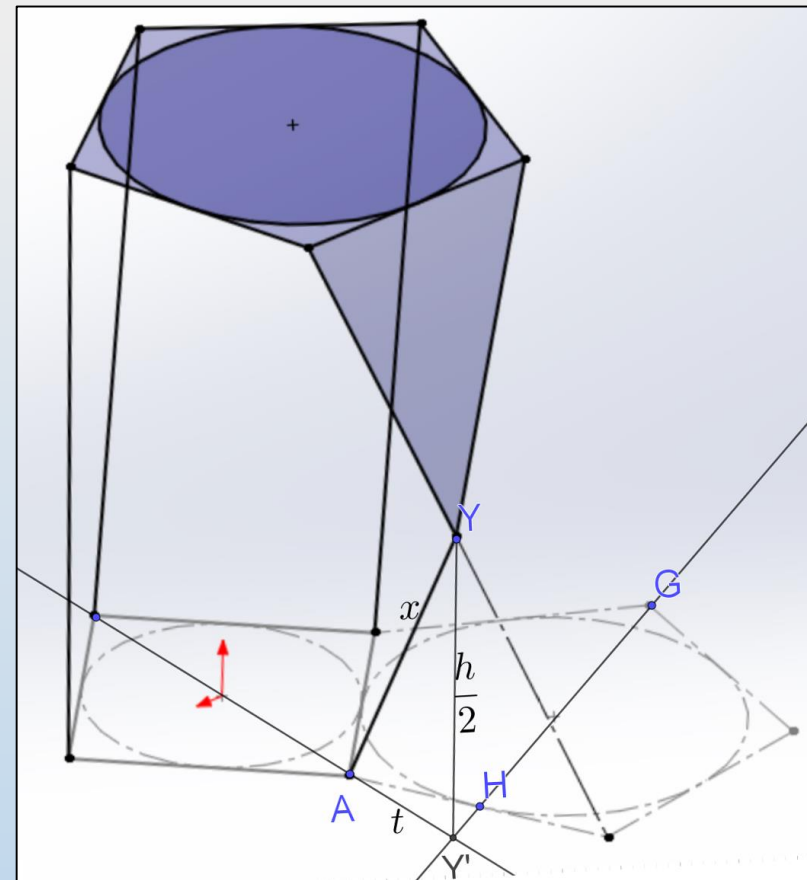
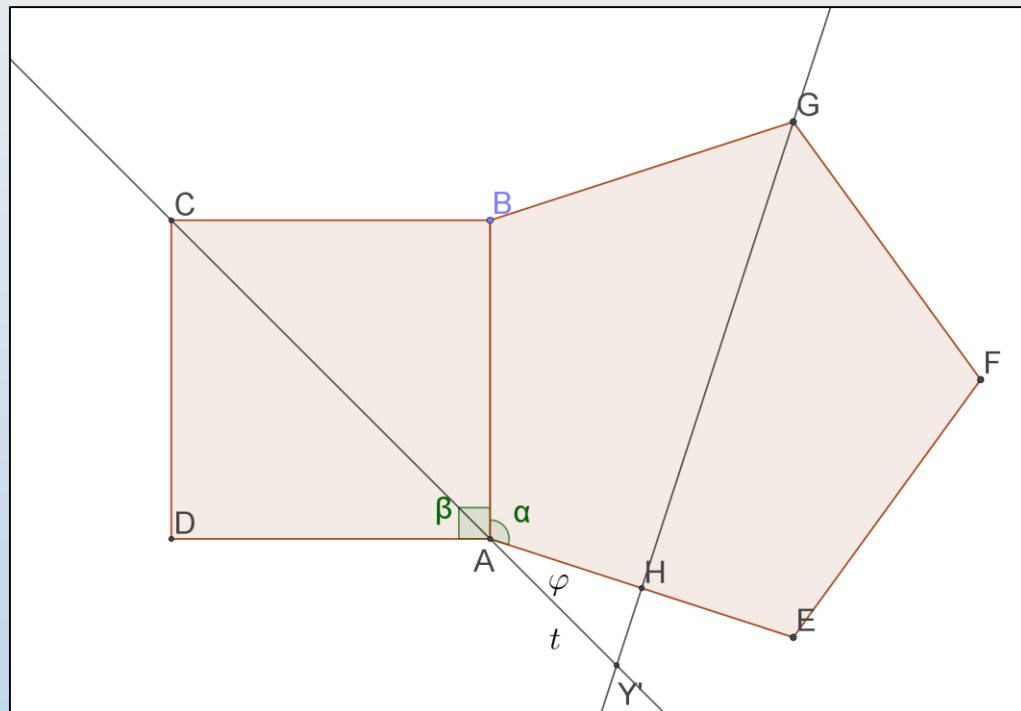


We can form the circle, because the equation $k \cdot 30 = 360$ is solvable in N : we get the regular twelve-angle

Because of the different angles of inclination and different lengths of the legs of scutellum different scutoids do not complement one another in the space between two surfaces.



General formula for angle $\varphi = \angle HAY'$



$$\alpha = \frac{(n-2) \cdot 180}{n}$$

$$\beta = \frac{(n-3) \cdot 180}{n-1}$$

$$\varphi = 180^\circ - \alpha - \frac{\beta}{2}$$

$$\varphi(n) = \frac{(n^2 - 7n + 4)}{(1-n)n} \cdot 90^\circ$$

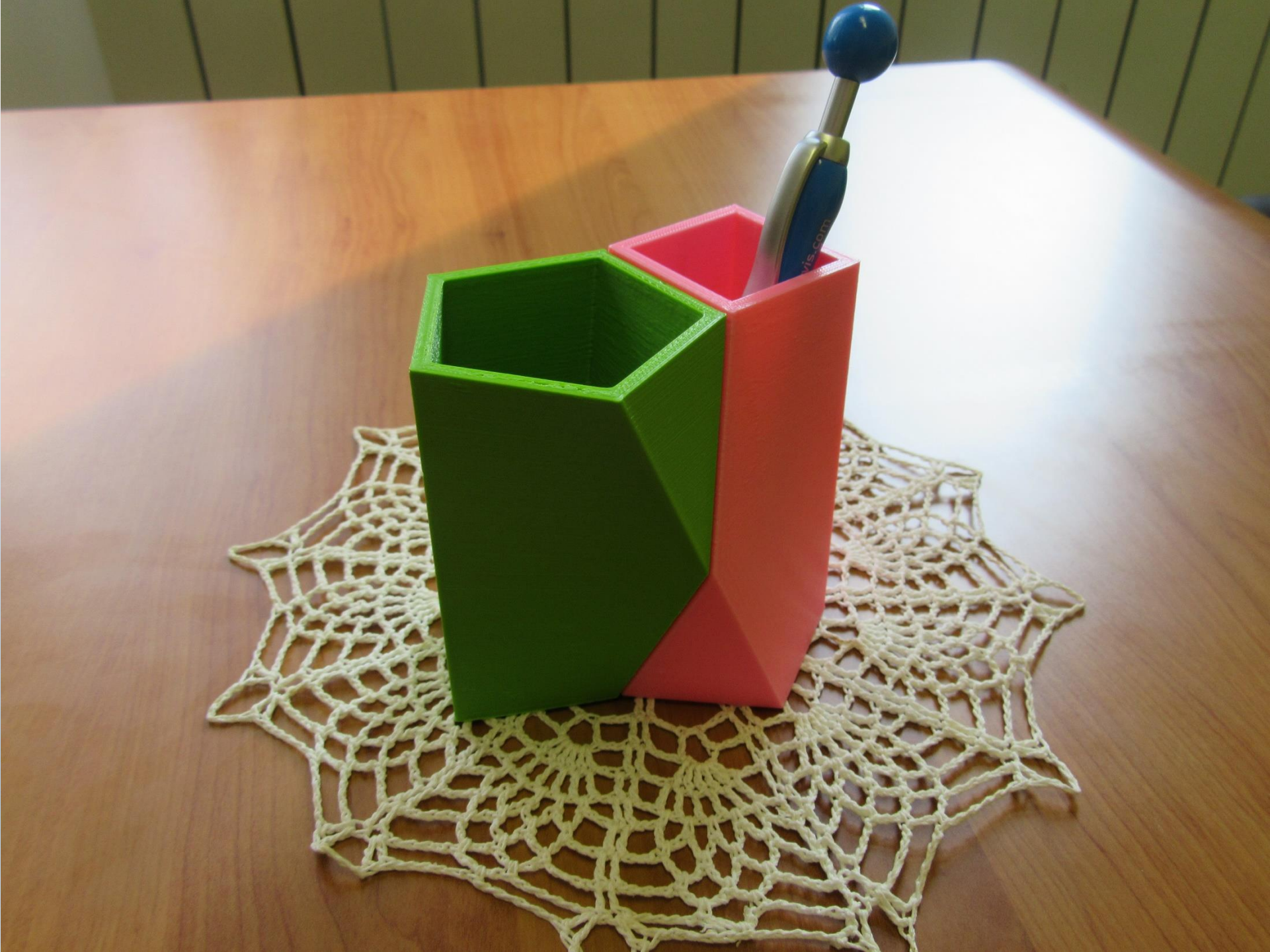
$$t = \frac{a}{2 \cdot \cos \varphi}$$

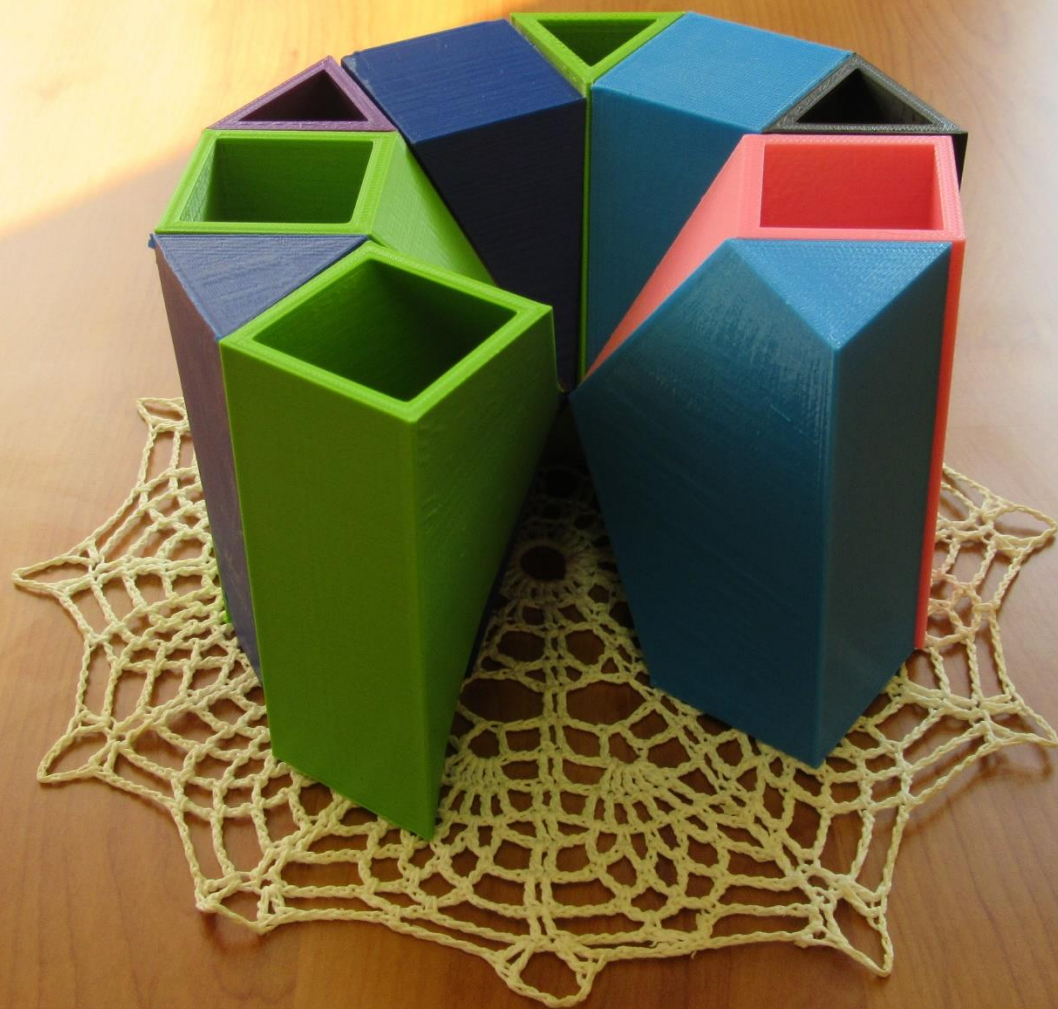
$$x^2 = \left(\frac{h}{2}\right)^2 + t^2$$

Possible scope of usage:

- when modeling the cells in the epithelial tissues
- in design
- in art
- as decorative elements
- maybe even in architecture
- as a creative toy



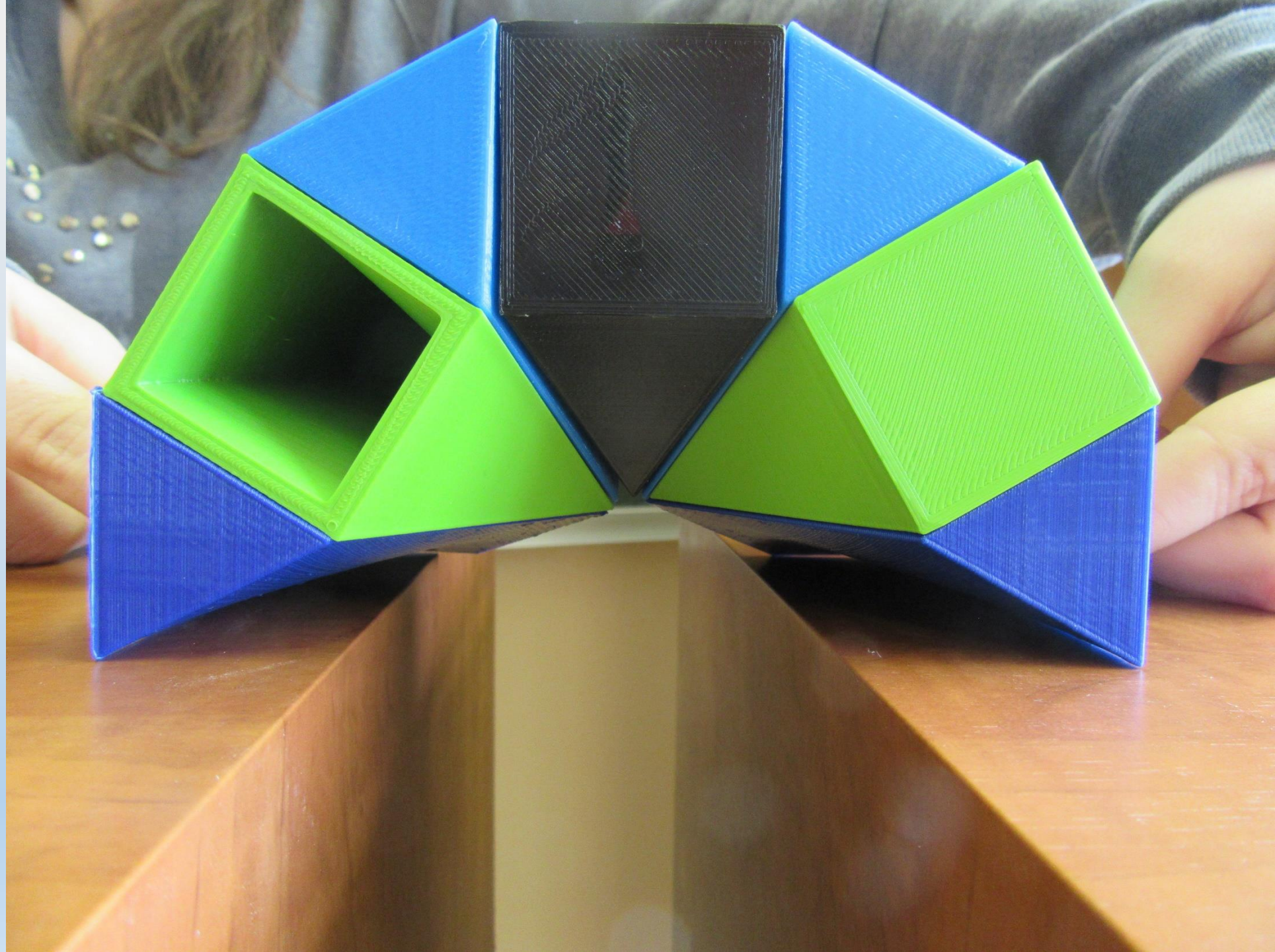


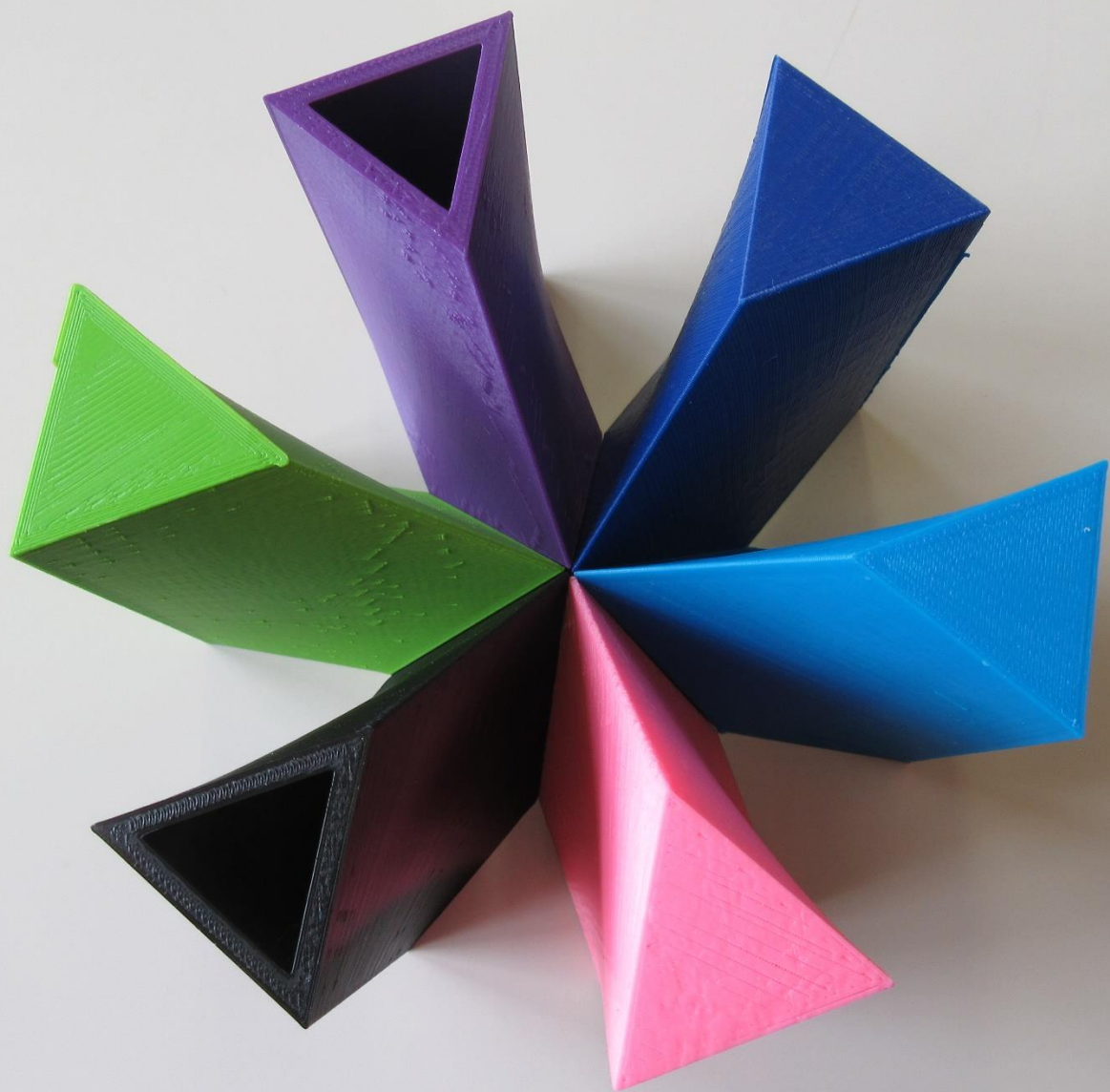








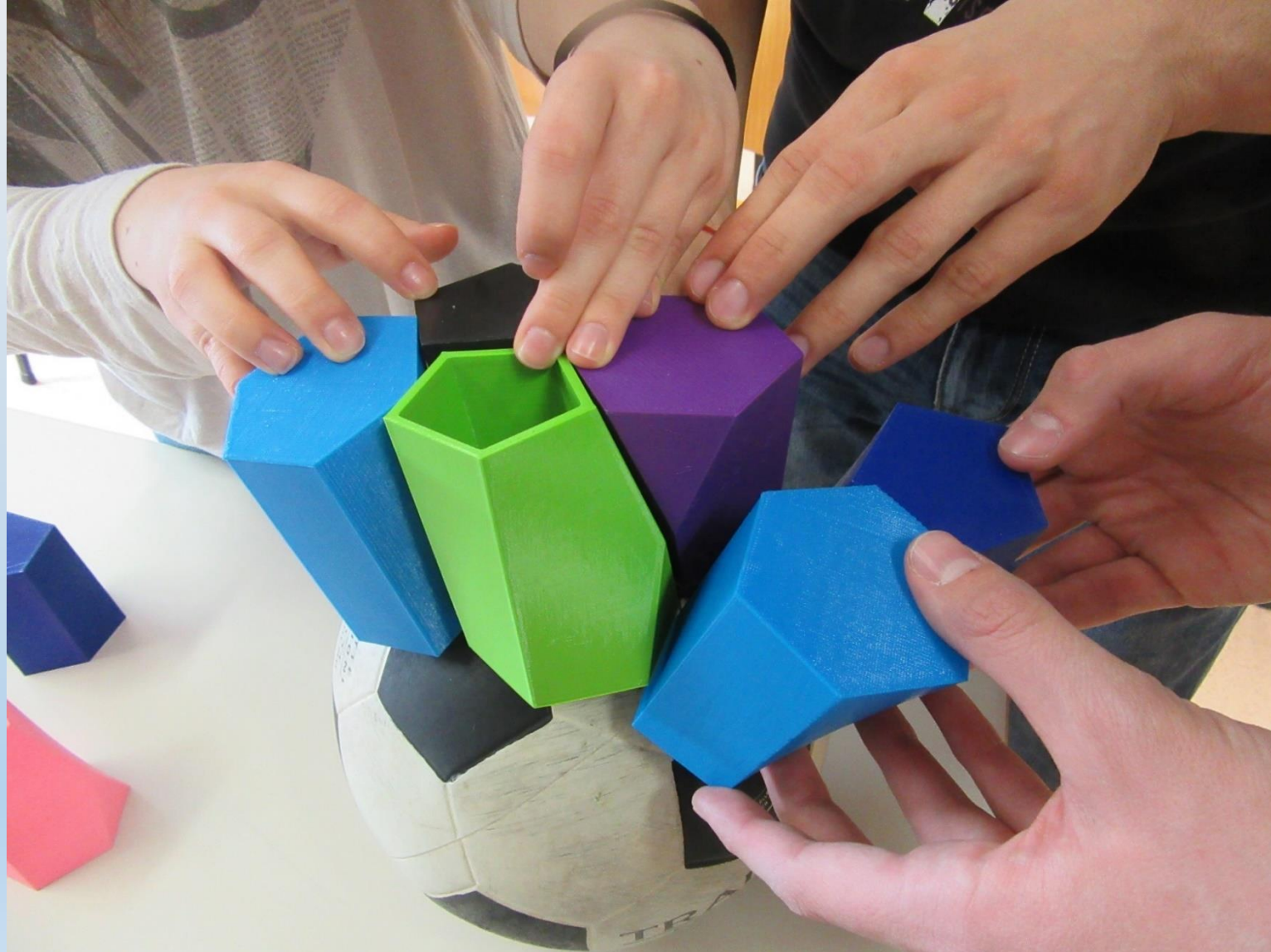




ACTIVITIES FOR CHILDREN (Set: two of 3-4, two of 45, two of 56)

1. From the multitude of scutoids pick those, who have the same base surface. Name the surface.
2. Put together the same kind of scutoids, so they combine completely. How many possibilities are there?
3. Take two sets of scutoids. Form chains of the same scutoids.
4. Take two sets. Form arches with the same kind of scutoids. In which cases can you make a circle?
5. Put together the same kinds of scutoids, so they complement each other. Is this even possible?
6. Take one set. From all three pairs of scutoids build a tower, so two scutoids touch each other with the same base surfaces. Find at least two possibilities.
7. Form buildings.
8. Draw the network of scutoids and form a geometric solid.

VIEW FORWARD





THANK YOU FOR YOUR ATTENTION